Lorentz Transformations for Laser Neutralization of H- Beams

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The purpose of this Note is to review the Lorentz transformations for laser stripping of relativistic H- beams. The binding energy of the "outer" electron is 0.75 eV, and the maximum photoneutralization cross section is about 4E-17 cm^2 at 1.5 eV. The neutralization yield is above 2E-17 cm^2 between about 1 and 3 eV.

The Lorentz transformation from the lab frame (unstarred) to the beam bunch rest frame (starred), with the forward (zero degree) direction in the lab being head-on collisions, is given by

$$\begin{matrix} E_\nu^* \\ p_L^* \end{matrix} = \begin{bmatrix} \gamma & +\beta\gamma \\ +\beta\gamma & \gamma \end{bmatrix} \begin{matrix} E_\nu \\ p_L \end{matrix} \qquad \text{and} \quad p_T^* = p_T \qquad [1]$$

where E_ν is the photon energy, p_L and p_T are the longitudinal and transverse photon momenta in energy (i.e., pc) units, and β and γ are the Lorentz parameters of the H- beam. For photons, the longitudinal momentum p_L may be written

$$p_L = E_\nu \cos\theta \quad and \quad p_L^* = E_\nu^* \cos\theta^* \qquad [2]$$

where θ and θ^* are the photon beam angles relative to the z axis in the lab and in the bunch rest frames respectively.

The Lorentz transformation from the beam bunch rest frame to the lab frame is given by

$$\begin{matrix} E_\nu \\ p_L \end{matrix} = \begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix} \begin{matrix} E_\nu^* \\ p_L^* \end{matrix} \qquad [3]$$

The laser photon energy in the bunch rest frame is thus given by

$$E_\nu^* = \gamma E_\nu \left[1 + \beta\cos\theta\right]. \qquad [4]$$

The laser photon energy boost factor (sometimes called the gamma boost) is then

$$E_\nu^* \Big/ E_\nu = \gamma\left[1 + \beta\cos\theta\right]. \qquad [5]$$

The angle transformations between the lab frame and the bunch rest frame are given by

$$\cos\theta^* = \frac{\beta\gamma + \gamma\cos\theta}{\gamma + \beta\gamma\cos\theta} \qquad \text{and} \quad \cos\theta = \frac{-\beta\gamma + \gamma\cos\theta^*}{\gamma - \beta\gamma\cos\theta^*} \qquad [6]$$

The following table presents Lorentz transformation values for laser stripping of a 1-GeV H- beam (β=0.8749). The laser photon beam angle is shown in both the lab frame and the center-of-mass (CM) frame, and the optimum laser photon energy and wavelength are shown for a 1.5-eV photon in the CM frame. 0° corresponds to head-on collisions. Note that a lab angle of 151° corresponds to 90° in the CM system, and 90° in the lab corresponds to 29° in the CM system. The optimum lab angle for a Nd:YAG laser is about 115°.

Laser angle (degrees)		Boost	Optimum laser photons	
Lab frame	CM	factor	energy (eV)	wavelength (microns)
0	0.0	3.87	0.39	3.198
5	1.3	3.86	0.39	3.193
10	2.6	3.84	0.39	3.176
15	3.9	3.81	0.39	3.148
20	5.2	3.76	0.40	3.108
25	6.6	3.70	0.41	3.059
30	7.9	3.63	0.41	2.998
35	9.3	3.54	0.42	2.929
40	10.7	3.45	0.43	2.849
45	12.2	3.34	0.45	2.761
50	13.7	3.23	0.47	2.665
55	15.3	3.10	0.48	2.562
60	17.0	2.97	0.51	2.452
65	18.7	2.83	0.53	2.337
70	20.5	2.68	0.56	2.216
75	22.4	2.53	0.59	2.092
80	24.5	2.38	0.63	1.965
85	26.6	2.22	0.68	1.836
90	29.0	2.06	0.73	1.706
95	31.5	1.91	0.79	1.576
100	34.2	1.75	0.86	1.447
105	37.2	1.60	0.94	1.320
110	40.5	1.45	1.04	1.195
115	44.1	1.30	1.15	1.075
120	48.2	1.16	1.29	0.960
125	52.8	1.03	1.46	0.850
130	58.0	0.90	1.66	0.747
135	63.9	0.79	1.91	0.651
140	70.7	0.68	2.20	0.563
145	78.7	0.59	2.56	0.483
150	87.9	0.50	3.00	0.413
155	98.7	0.43	3.51	0.353
160	111.4	0.37	4.08	0.303
165	126.0	0.32	4.69	0.264
170	142.6	0.29	5.25	0.236
175	160.8	0.27	5.66	0.219
180	180.0	0.26	5.81	0.213