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INSTABILITIES IN THE SNS*

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Abstract

The 2MW Spallation Neutron Source (SNS) will have a D.C. beam current of 40 A at extraction, making it one of the worlds most intense accelerators. Coherent instabilities are a major concern and efforts to predict beam behavior are described.

1 INTRODUCTION

For 2 MW operation the SNS will accumulate 2×10^{14}, 1 GeV protons over 1 ms via charge exchange injection. The machine circumference is 220 m with a transition energy of $\gamma_t = 4.9$ and betatron tunes around 5.8[1]. The baseline design calls for natural chromaticities ~ -1, but chromatic control is likely in the final design. The machine impedance below 200 MHz has been characterized[2, 3], and measurements of the extraction kicker impedance are in progress.

2 LONGITUDINAL INSTABILITIES

Two methods have been used to characterize longitudinal stability in the SNS. First assume a coasting beam with a rectangular energy distribution and do first order pertubation theory on the Vlasov equation with

$$I = I_0 + I_1 \exp(in(\theta - \omega_0 t) - i\Omega t).$$

The dispersion relation is given by,

$$\frac{\Omega^2}{\omega_0^2 n^2} = \hat{v}^2 + i\frac{qI_0\eta}{2\pi E_0\beta^2}\frac{Z_\parallel(\Omega + n\omega_0)}{n} \tag{1}$$

where $\eta = -0.193$ is the frequency slip factor, $E_0 = \gamma mc^2$, q is the proton charge, and $\hat{v} = |\eta|\max(E - E_0)/E_0\beta^2 \approx 1 \times 10^{-3}$. With $I_0 = 40$A the terms on the right had side are equal for $Z_\parallel/n = i650\Omega$. The space charge impedance is $Z_\parallel/n = i150\Omega$ and dominates the magnitude of the impedance. Taking the square root of eq(1) and assuming the second term is small compared to the first yields

$$Im(\Omega) \approx \frac{\omega_0}{2\hat{v}}\frac{eI_0\eta}{2\pi E_0\beta^2}Re(Z_\parallel). \tag{2}$$

When all objects other than the extraction kicker are included in the impedance budget the growth rate of the most unstable mode below 700 MHz is $\lesssim 2$/ms, which is benign. Near the cutoff frequency for transverse magnetic microwave propagation ≈ 1 GHz, codes such as ABCI [6] predict strong, narrow, resonant impedances. These will be considered in the future.

Instabilities of the sort predicted by eq (1) and (2) are rare or nonexistent below transition. The reason can be inferred from a simple bunched beam model. Assume a “brick wall” barrier bucket rf system. The bunch has length τ_b (radians) and peak current I_0. The particles undergo perfect reflection at the edges of the bunch and the impedance is given by $Z = R - i\omega L$. Use the machine azimuth θ as the time-like variable. Let $\tau = \omega_0 t - \theta$ and $v = d\tau/d\theta$ be the dynamical variables. Assume the unperturbed phase space density is given by

$$\Psi_0(\tau, v) = \frac{1}{2\hat{v}\tau_b}$$

for $0 < \tau < \tau_b$ and $|v| < \hat{v}$, and zero otherwise. This neglects the effect of R on the unperturbed distribution. Assume a solution to the Vlasov equation of the form

$$\Psi(\tau, v, \theta) = \Psi_0(\tau, v) + e^{-iQ\theta}\Psi_1(\tau, v).$$

The pertubation satisfies

$$-iQ\Psi_1 + v\frac{\partial\Psi_1}{\partial\tau} = F_1(\tau)\left(\delta(v - \hat{v}) - \delta(v + \hat{v})\right)$$

where

$$\Psi_1(\tau, v) = \Psi_+(\tau)\delta(v - \hat{v}) + \Psi_-(\tau)\delta(v + \hat{v}) \tag{3}$$

$$F_1(\tau) = \kappa\left(R + \omega_0 L\frac{d}{d\tau}\right)(\Psi_+(\tau) + \Psi_-(\tau)) \tag{4}$$

and

$$\kappa = \frac{-\eta q I_0}{4\pi\hat{v}\beta^2 E_0}$$

The equations for $\Psi_+(\tau)$ and $\Psi_-(\tau)$ are first order with constant coefficients. To solve these equations let $U - iV = \kappa(R - i\omega_0 L)$, $S(\tau) = \Psi_+(\tau) + \Psi_-(\tau)$, and $D(\tau) = \Psi_+(\tau) - \Psi_-(\tau)$. Then

$$-iQS + \hat{v}\frac{dD}{d\tau} = 0, \tag{5}$$

$$-iQD + (\hat{v} - 2V)\frac{dS}{d\tau} = 2US. \tag{6}$$

For perfect reflection at $\tau = 0$ and $\tau = \tau_b$ the boundary conditions are $D(0) = D(\tau_b) = 0$, so

$$D(\tau) = \exp(\lambda_+\tau) - \exp(\lambda_-\tau).$$

Equations (5) and (6) give

$$\lambda_\pm = \frac{U}{\hat{v} - 2V} \pm \sqrt{\frac{U^2}{(\hat{v} - 2V)^2} - \frac{Q^2}{\hat{v}(\hat{v} - 2V)}}$$

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The boundary condition at τ_b gives $\lambda_+ - \lambda_- = 2\pi ik/\tau_b$ with $k \neq 0$ an integer. Solving for $\Omega = \omega_0 Q$ gives

$$\Omega^2 = \omega_0^2 \hat{v}(\hat{v} - 2V) \left\{ \frac{k^2\pi^2}{\tau_b^2} + \frac{U^2}{(\hat{v} - 2V)^2} \right\} \quad (7)$$

The right hand side of (7) is positive as long as $\hat{v} > 2V$ or

$$-\frac{qI_0\eta}{2\pi E_0 \beta^2}\omega_0 L < \hat{v}^2.$$

This is very different from the coasting beam result. For the SNS space charge dominates $(L < 0)$ and we are below transition $\eta < 0$ so no amount of resistance makes the beam unstable. For a resonator impedance there are 4 coupled ODEs and numerical techniques apply [5]. Figure 1 shows the eigentunes for a wake potential given by

$$W(\tau) = W_0(1 - 10\pi\tau/\tau_b)e^{-10\pi\tau/\tau_b},$$

corresponding to a critically damped resonantor with a frequency of 8 MHz in SNS. The tunes are in units of $2\tau_b/\hat{v}$ and are plotted versus the tune shift obtained by assuming $D(\tau) = \sin(\pi\tau/\tau_b)$. The system appears stable for *all* values of W_0, not just those shown. This too is a curious result and independent confirmation would be appreciated.

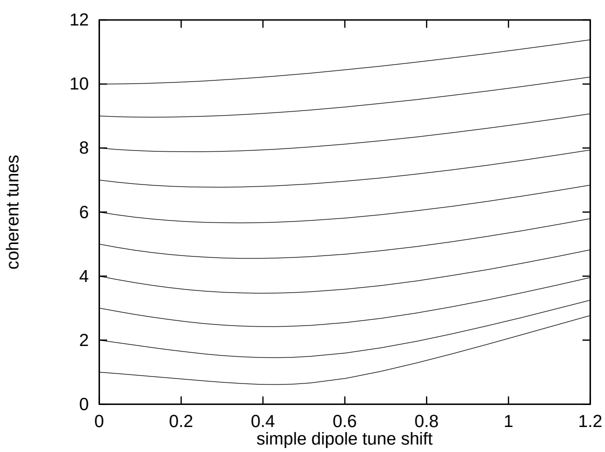

Figure 1: Exact coherent tunes versus tune shift for the dipole mode in the weak coupling approximation

For long range wakefields the residual fields from previous turns must be included in the equations of motion. Studies in this direction continue and growth rates of order the synchrotron frequency are expected. Since the entire SNS cycle is about half a synchrotron period no serious instabilities are expected.

The final type of longitudinal stability is related to the rf system. With large beam current the coupling between the beam, cavity and power amplifier is a major consideration in rf design. For SNS we have simulated the effects of beam loading with realistic amplifier passbands and delays. As of now the delays are large enough to keep the feedforward and feedback amplifiers outside the tunnel. The amplifiers will go into the tunnel if needed.

3 TRANSVERSE INSTABILITIES

Both coasting and bunched beam approximations have been used to study transverse stability. The space charge tune shift reaches ≈ 0.2 while $\Delta p/p \lesssim 0.01$. For a coasting beam with a parabolic energy distribution a normalized chromaticity ≈ -16 is needed to Landau damp low frequency oscillations[7]. Such a chromaticity would have a strong impact on dynamic aperture so no Landau damping due to chromaticity will be assumed. A similar statement applies to octupoles. Landau damping due to frequency slip becomes effective for mode numbers $n \gtrsim 4 \times 0.2/\hat{v} \approx 800$. This frequency is above cutoff for transverse electric microwave propagation $\approx$ 800 MHz where the transverse impedance may have narrow resonances. Therefore, a cold coasting beam dispersion relation is appropriate and the growth rate is[7]

$$Im(\Omega) = \frac{qcI_{peak}Re(Z_\perp(n - Q_\beta))}{4\pi E_0 Q_\beta} = 2.12 I_A Z_K \mathrm{s}^{-1}, \quad (8)$$

where $Q_\beta \approx 5.8$, I_A is the current in amps, Z_K is the transverse impedance in kΩ/m, and $n > 0$ for instability. For the narrow band resistive wall growth rate $I_A = 40$ and $Im(\Omega) = 1.1/\mathrm{ms}$, which is benign. The stripline beam position monitors (BPMs) have a transverse resistance of $25\mathrm{k}\Omega/\mathrm{m}$ near 100 MHz and a large bandwidth. Taking $I_A = 100$ gives $Im(\Omega) = 5.3/\mathrm{ms}$. This is a large growth rate, but the wide bandwidth of the BPM impedance requires that the finite bunch length be taken into account.

Bunched beam stability calculations have been done assuming an air bag longitudinal distribution in a square well [4, 5]. The impedance budget includes the resistive wall, BPMs, extraction kicker, bellows, and transitions. The high frequency narrow band resonances associated with the latter were ignored. The extraction kicker impedance was modeled using a slightly modified version of the Nassibian Sacherer formula[8].

$$Z_\perp(\omega) = \frac{-iNcL}{g^2}\frac{Z_g/L}{Z_g/L - i\omega},$$

where N is the number of kicker modules, L is the inductance per module, g is the aperture in the kick direction, and Z_g is the generator impedance of the pulse forming network. This formula differs from [8] in that $Z_\perp \to 0$ as $\omega \to \infty$ here. Both have identical expressions for $Re(Z_\perp)$.

Table 1 shows the growth rate of the most unstable mode as a function of space charge tune shift and kicker generator impedance. The growth rates are for 2×10^{14} protons and should be multiplied by 0.5ms to obtain the number of e-folding times in the cycle. In the worst case there are 2.5 e-folding times in the SNS cycle. An initial offset of 1mm yields 1.2cm at extraction.

4 ELECTRON PROTON INSTABILITY

Very fast, high frequency, transverse instabilities have been observed in the Los Alamos PSR and the AGS Booster.

Table 1: Transverse bunched beam growth rates as a function of space charge tune shift and kicker generator impedance.

ΔQ_{sc} 1	$Z_g = \infty$ $10^3\mathrm{s}^{-1}$	$Z_g = 500\Omega$ $10^3\mathrm{s}^{-1}$	$Z_g = 50\Omega$ $10^3\mathrm{s}^{-1}$	$Z_g = 5\Omega$ $10^3\mathrm{s}^{-1}$
0.00	1.27	1.41	2.01	3.85
0.05	1.37	1.52	4.58	3.69
0.10	1.37	3.14	4.87	3.70
0.15	1.37	4.27	4.99	3.70
0.20	1.37	4.99	5.05	3.70

The e-folding times (~ 10 turns) require a transverse resistance or order $1\mathrm{M}\Omega/\mathrm{m}$ which is significantly larger than can be accounted for by the lattice. Additionally, the frequency of the instability depends strongly on beam parameters like betatron frequency ω_β, which requires a broad band impedance. An alternate driving mechanism is the electrostatic interaction between the proton beam and electrons within the beam pipe[9, 10, 11, 12].

Assume a coasting proton beam of radius a with vertical offset

$$Y_p(\theta, t) = \hat{Y}_p \exp[\, in(\theta - \omega_0 t) + i(\omega_\beta + \delta\omega_\beta)t]$$

where θ is the machine azimuth and $\delta\omega_\beta$ is the betatron frequency shift. Let an electron cloud be trapped by the beam with vertical offset

$$Y_e(\theta, t) = \hat{Y}_e \exp[in\theta - i(\omega_e + \delta\omega_e)t]$$

where

$$\omega_e = \sqrt{\frac{eIZ_0}{2\pi\beta m_e a^2}}$$

is the incoherent transverse frequency for electrons trapped within the proton beam. Of course $\delta\omega_e = -\delta\omega_\beta + (n\omega_0 - \omega_\beta - \omega_e)$. The equations of motion are given by

$$\ddot{Y}_p = -\omega_\beta^2 Y_p + \omega_p^2(Y_e - Y_p) \quad (9)$$

$$\ddot{Y}_e = \omega_e^2(Y_p - Y_e) \quad (10)$$

where

$$\omega_p^2 = f\frac{m_e}{\gamma m_p}\omega_e^2$$

with f being the fractional neutralization due to the electrons. Assuming $\delta\omega_\beta \ll \omega_\beta$ and $\delta\omega_e \ll \omega_e$

$$2i\omega_\beta\delta\omega_\beta Y_p = \omega_p^2 Y_e, \quad -2i\omega_e\delta\omega_e Y_e = \omega_e^2 Y_p$$

Solving for $\delta\omega_\beta$ yields

$$\delta\omega_\beta = \frac{\Delta\omega}{2} \pm \sqrt{\left(\frac{\Delta\omega}{2}\right)^2 - \frac{\omega_p^2\omega_e}{4\omega_\beta}}$$

where $\Delta\omega = n\omega_0 - \omega_\beta - \omega_e$. For unstable conditions the ratio of proton to electron amplitudes is given by

$$\left|\frac{\hat{Y}_p}{\hat{Y}_e}\right| = \frac{\omega_p}{\sqrt{\omega_e\omega_\beta}}$$

Since $|\Delta\omega| \le \omega_0/2$ for some n the beam will be unstable if $\omega_p^2 \ge \omega_0^2\omega_\beta/4\omega_e$ which gives $\hat{Y}_p \sim \omega_0\hat{Y}_e/\omega_e \ll \hat{Y}_e$.

Electrons with $\hat{Y}_e = b$ (the pipe radius) will be created by the familiar slow loss mechanisms. As f increases the beam goes unstable and the electron amplitudes grow. An electron striking the beam pipe leads to secondary emission which can cause an electron cascade and beam loss. A key parameter is the kinetic energy the electron has when it strikes the wall. This is easily estimated by assuming the electron grazes the wall on one oscillation and hits it on the next. If only one side of the vacuum chamber is involved the electron velocity on impact is given by

$$v_e = b\sqrt{4\pi\omega_e\omega_I}\left(1 + O(\sqrt{\omega_I/\omega_e})\right)$$

where $\omega_I = Im(\delta\omega_e)$. Assuming $\Delta\omega = 0$, the electron kinetic energy when striking the wall is

$$KE_e = \pi m_e(\omega_e b)^2\sqrt{\frac{fm_e}{\gamma m_p}\frac{\omega_e}{\omega_\beta}}. \quad (11)$$

Table 2 summarizes the observations of the instability in the AGS Booster and PSR. For $f = 1\%$ the electrons hit the wall with an energy greater than 100eV, which results in a secondary emission coefficient greater than one for most metals. With $f = 1$ both machines have $|\hat{Y}_p/\hat{Y}_e| < 0.1$ so strong multipactoring is required for fast beam loss.

Table 2: Coasting beam instability parameters

machine	$\omega_e/2\pi$	$\omega_\beta/2\pi$	b	γ	$KE_e/\sqrt{f}$
	MHz	MHz	cm	1	keV
PSR	100	6.0	5	1.85	1.24
Booster	80	4.1	6	1.21	1.52

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