Mathcad Simulation of Mismatched CCL BPM Electrodes
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This simulation compares the signal ouput of impedance-mismatched BPM electrodes in both the time and frequency domains. In the time domain, if the electrode impedance is mismatched to the output load, reflections occur at the interface, creating follow-on pulses. In the frequency domain, the power spectrum chnages, increasing for impedances over 50 ohms, and decreasing for impedances under 50 ohms.

==========Time domain simulation==========

A 45-degree, 4 cm long, 50 ohm SNS CCL electrode is compared with a mismatched 60 degree, 4 cm, 40 ohm electrode.

==========constants==========

$L := 4$ electrode length, cm

$c := 2.997924 \cdot 10^{10}$ speed of light

$dt := \frac{2 \cdot L}{c}$ round trip signal delay

$s := 40 \cdot 10^{-12}$ rms beam pulse length

These parameters relate to the impedance of the pickup as seen by the beam, and the refrection and transmission at the electrode to output impedance mismatch.

$F(Z) := \frac{Z \cdot 50}{Z + 50}$ input impedance seen by beam. It is the parallel resistance of the output and the electrode.

$R(Z) := \frac{50 - Z}{50 + Z}$ reflection mismatch coefficient at the output end. The reflection mismatch coefficient at the shorted end of the electrode is -1.

$T(Z) := \frac{100}{Z + 50}$ transmission mismatch coefficient, for signals going from the electrode to the output. T(50)=1.

$G(t, m) := \exp\left[-0.5 \cdot \left(\frac{t - m \cdot dt}{s}\right)^2\right]$ Gaussian pulse shape with delay due to transit time on electrode.

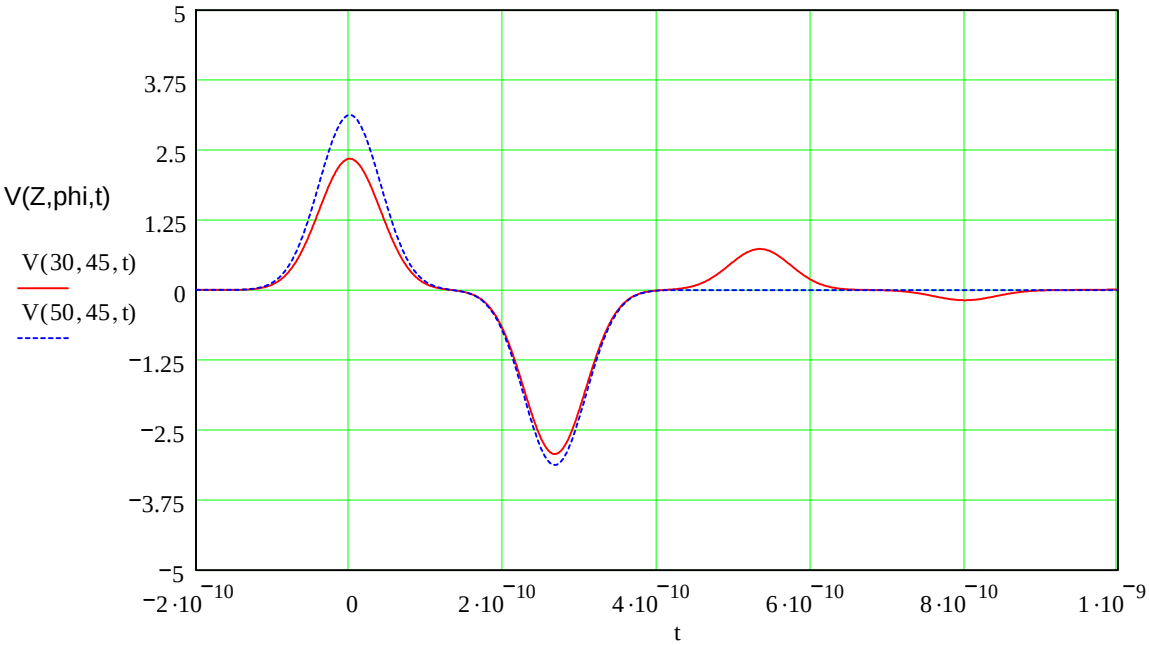
This is the voltage waveform at the output due to the beam coupling to the electrode and to multiple reflections on the electrode. Phi is the aximuthal width of the electrode. Three reflections are included in this model.

$$V(Z, phi, t) := \frac{phi}{360} \cdot F(Z) \cdot \left(G(t,0) - T(Z) \cdot G(t,1) + R(Z) \cdot T(Z) \cdot G(t,2) - R(Z)^2 \cdot T(Z) \cdot G(t,3) + R(Z)^3 \cdot T(Z) \cdot G(t,4)\right)$$

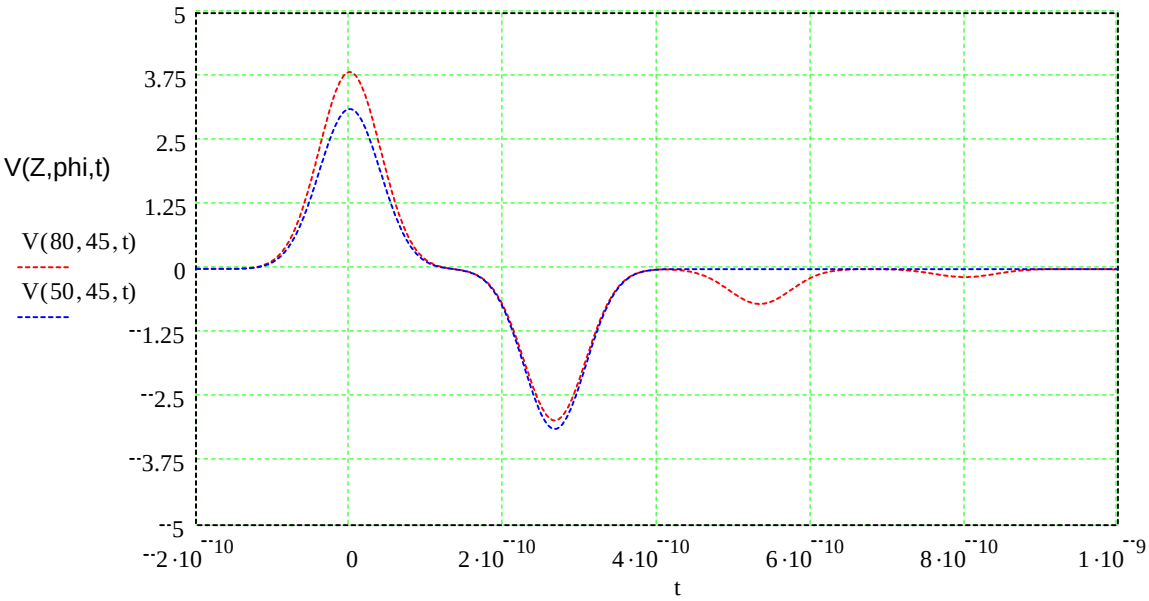
Plot time range

$t := -200 \cdot 10^{-12}, -199 \cdot 10^{-12} .. 1 \cdot 10^{-9}$

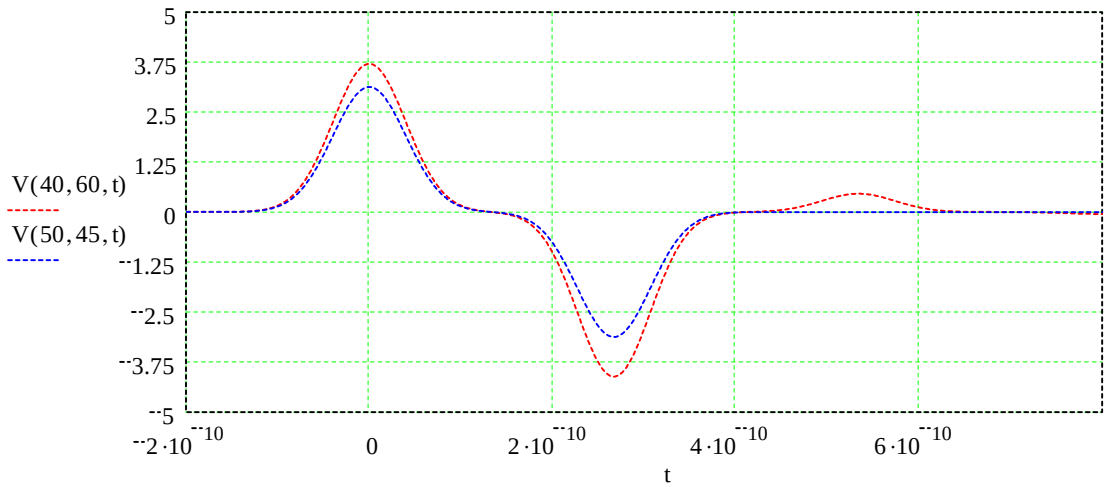
This plot compares a 30 ohm 45 degree electrode to a 50 ohm 45 degree electrode. Note that the reflections alternate polarity.



This plot compares a 80 ohm 45 degree electrode to a 50 ohm 45 degree electrode. Note that the reflections do not alternate polarity.



This plot compares a 40 ohm, 60 degree electrode to a 50 ohm, 45 degree electrode.



========== Frequency domain analysis ==========

$f := 5, 10 .. 5000 \qquad w(f) := 2 \cdot \pi \cdot f \cdot 10^{6}$

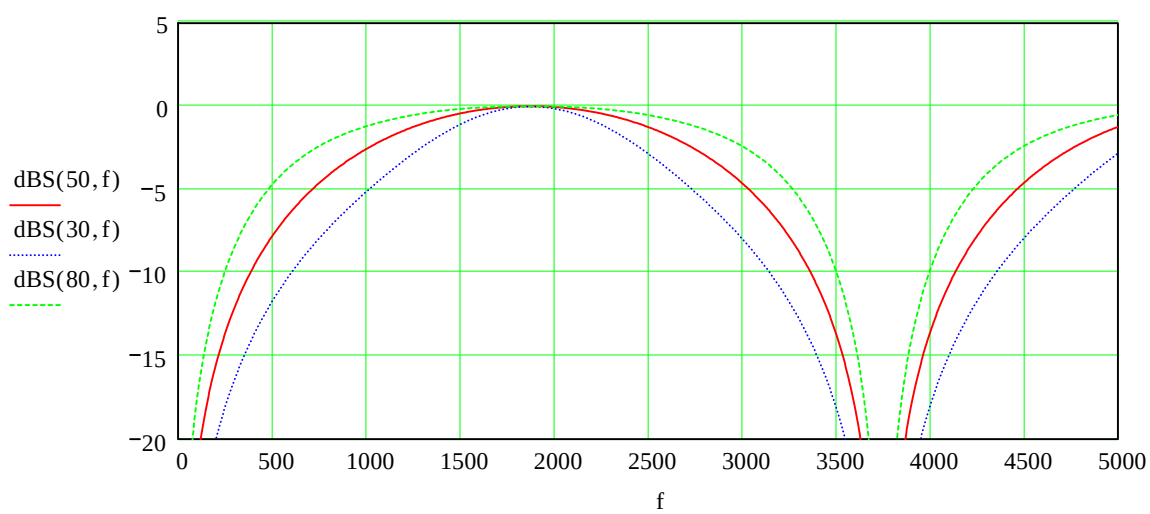
$E(f, m) := e^{-j \cdot w(f) \cdot m \cdot dt}$ phase delay, exponential notation

Frequency-domain representation of multiple reflections.

$$S(Z, f) := 0.02 \cdot F(Z) \cdot \left(1 - T(Z) \cdot E(f, 1) + R(Z) \cdot T(Z) \cdot E(f, 2) - R(Z)^{2} \cdot T(Z) \cdot E(f, 3) + R(Z)^{3} \cdot T(Z) \cdot E(f, 4)\right)$$

$dBS(Z, f) := 20 \log(|S(Z, f)|)$ dB scale

Comparison of power vs. frequency for 30, 50, and 80 ohm electrodes. Note that even though the 80 ohm electrode is mismatched, its power output is highest.



The signal processing frequency in the SNS CCL linac is 402.5 MHz.

$dBS(30, 402) = -13.7$ dB $\qquad dBS(50, 402) = -9.6$ dB $\qquad dBS(80, 402) = -6.2$ dB