SNS-NOTE-DIAG-39

Signal Analysis for the Mode Locked Laser used as a Laser-Wire for Modulated and Unmodulated Sources

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Abstract: The mode locked laser has been discussed as a tool for determining the transverse profile of the beam at SNS. An analysis for the signals and spectrum of the resulting current of the mode locked laser is presented.

I. Introduction.

The mode locked laser has been discussed as a tool measuring the transverse profile of the SNS beam. The basic functionality of the laser is presented in Fig. 1.[1] It is assumed that a train of micropulses progress along the linac. The micropulses are separated in time by the RF-period, T_{RF}. Every T_L seconds, it is assumed that the laser is switched on and the laser light frees some of the electrons on a single micropulse. In effect, this reduces the current on a single micropulse. It is assumed that the process reoccurs ad infinitum.

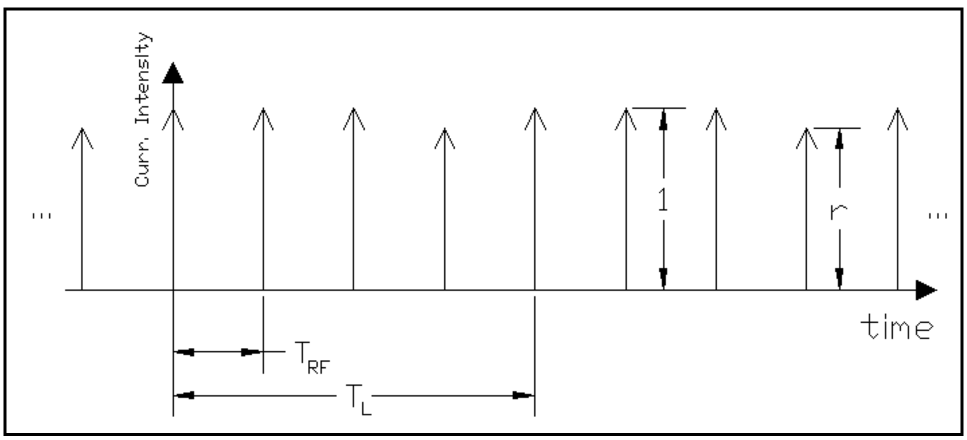

Figure 1. An infinite train of uniform micropulses is periodically modulated by a laser. The laser reduces the current in the modulated micropulse. The RF period is T_{RF}, and the period with which the laser pulses is T_L. It is assumed that the current intensity of the unperturbed micropulses is unity magnitude, while the perturbed micropulse has magnitude "r."

[1]A. Aleksandrov, private communication.

SNS is managed by UT-Battelle, LLC, under contract DE-AC05-00OR22725 for the U.S. Department of Energy. SNS is a partnership of six national laboratories: Argonne, Brookhaven, Jefferson, Lawrence Berkeley, Los Alamos, and Oak Ridge.

Since it is assumed that this process reoccurs, one can think of the train of micropulses as a periodic current source. The spectrum of this current source is analyzed and the results are presented for viability as a tool for the proposed laser-wire.

It has also been discussed that the source may modulate the current train. This modulation is also analyzed and results presented.

II. Analysis of Unmodulated Current Train.

Consider the bunch structure depicted in Fig. 1. To make the mathematics easier, assume that the current sources are impulses of current. A more realistic current distribution can be used, namely Gaussian distributions, but the mathematics becomes messier while the final conclusion remains the same. The current distribution shown in Fig. 1 is described by:

$$I(t)=\sum_{n=0}^{p-1}\delta\left(t-nT_{RF}\right)+r\delta\left(t-pT_{RF}\right),$$

where p is the number of pulses between laser pulses (in Fig. 1, p=3), and $\delta(t)$ is the Dirac delta function. Since we assume that this general pulse train repeats itself, it is possible to study the harmonic spectrum of the current distribution I(t). Therefore,

$$I\left(t\right)=\sum_{j=-\infty}^{\infty}a_j e^{i\omega_j t},$$

where

$$\omega_j=\frac{2\pi j}{T_L}.$$

Using the standard techniques, it is easy to see that

$$a_j=\frac{1}{T_L}\int_0^{T_L^-}dtI\left(t\right)e^{-i\omega_j t}. \tag{1}$$

For j=0,

$$a_0=\frac{1}{T_L}\int_0^{T_L^-}dt\left(\sum_{n=0}^{p-1}\delta\left(t-nT_{RF}\right)+r\delta\left(t-pT_{RF}\right)\right)=\frac{p+r}{T_L}.$$

This equation shows that the DC component of the spectrum is dominated by the effects of the number of unperturbed micropulses, and is inversely proportional to the laser period T_L.

For $j \neq 0$,

$$a_j = \frac{1}{T_L}\left\{\int_0^{T_L^-} dt\left(\sum_{n=0}^{p-1}\delta(t-nT_{RF})e^{-i\frac{2\pi j}{T_L}t} + r\delta(t-pT_{RF})e^{-i\frac{2\pi j}{T_L}t}\right)\right\}$$

carrying out the integration,

$$a_j = \frac{1}{T_L}\left\{\sum_{n=0}^{p-1} e^{-i2\pi nj\tau} + re^{-i2\pi pj\tau}\right\}, \quad \tau = \frac{T_{RF}}{T_L}.$$

Note that the quantity τ is a fraction or rational number. The part in the summation is a simple geometric sum and is rewritten for $j\tau \notin \mathbb{N}$:

$$a_j = \frac{1}{T_L}\left\{\frac{1-e^{-i2\pi\tau pj}}{1-e^{-i2\pi\tau j}} + \frac{r\left(1-e^{-i2\pi\tau j}\right)e^{-i2\pi\tau pj}}{1-e^{-i2\pi\tau j}}\right\}. \tag{2}$$

If $j\tau \in \mathbb{N}$,

$$a_j = \frac{p+r}{T_L}. \tag{3}$$

Equation 3 is the identical dual to the zero-th harmonic described earlier, and relates to the natural RF-harmonic of the micropulse.

Note the following relationships:

$$T_L = (p+1)T_{RF}, \quad p+1 = \frac{1}{\tau}, \quad p = \frac{1-\tau}{\tau}. \tag{4}$$

Inserting Eq. 4 into Eq. 2, we see that

$$a_j = \frac{1-r}{T_L}\frac{1-e^{i2\pi\tau j}}{1-e^{-i2\pi\tau j}}. \tag{5}$$

The first term is a magnitude term and is dependant on the amount of current perturbed by the laser. The second term describes the phase behavior of the spectrum is solely dependant on the relative periodicity between the laser and the micropulses. Since τ is a rational quantity, the right side of Eq. 4 will take on discrete quantities.

Equation 5 also shows that the a successful frequency domain measurement relies on r<<1. This implies that the laser can efficiently bombard electrons free from the H^-.

The final result is as follows:

$$a_j = \begin{cases} \dfrac{p+r}{T_L} & \tau j \in \mathbb{N} \\ \dfrac{1-r}{T_L}\dfrac{1-e^{i2\pi\tau j}}{1-e^{-i2\pi\tau j}} & \tau j \notin \mathbb{N}. \end{cases} \tag{6}$$

III. Application to SNS Beam

It is necessary to analyze the micropulse beam in the SNS linac. A typical profile of the beam at the high energy section of the SNS linac has the following structure[2]:

$$I_{micro}(t) = \frac{I_b T_{RF}}{\sqrt{2\pi}\sigma_t} e^{-\frac{t^2}{2\sigma_t^2}},$$

where

$$I_b = 38\ mA \quad T_{RF} = 2.5\times10^{-9}\ \text{sec} \quad \sigma_t \approx 20\times10^{-12}\ \text{sec}.$$

This equation relates to the magnitude of the delta function current assumed in this paper. The total curren

The efficiency of the laser is on the order of 10^{-4}. Assume that every-other pulse is a laser perturbed pulse. This results in the following a_j:

$$a_{2j} = (0.038\ A)T_{RF}\frac{1+0.9999}{2T_{RF}} = 0.037998\ A$$
$$a_{2j+1} = (0.038\ A)T_{RF}\frac{1-0.9999}{2T_{RF}} = 3.8\times10^{-6}\ A.$$

This implies that the total power delivered to a 50Ω load for a particular harmonic is for the 2j

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[2]A. Aleksandrov, private communication.

mode is 72 mW=18.5dBm. The total power delivered to a 50Ω load for a particular harmonic of the 2j+1 mode is 722pW=-61.4 dBm.

One additional factor that is useful in this analysis is the method by which the signal is detected. It has been proposed to use a BPM for this measurement. A typical BPM is coupled rather loosely to the beam, and typical performance of a BPM is on the order of -20 dB. This says that it would only be possible to measure -81.4 dBm. While this number is certainly measurable with high-end commercially available equipment, it is necessary to add in any additional noise from the machine.

IV. Analysis of Modulated Current Train

Consider a source current having the following distribution:

$$I(t) = \left(1 + m\cos(\omega_s t)\right)\left\{\sum_{q=-\infty}^{\infty}\sum_{n=0}^{p-1}\delta\left(t - nT_{RF} - qT_L\right) + r\delta\left(r - pT_{RF} - qT_L\right)\right\}. \qquad (7)$$

This current distribution shows that the source is modulated by a factor of $\cos(\omega_s t)$, where the source modulation is assumed to be a function of the front end source. Since Eq. 7 is aperiodic, the Fourier-series approach is no longer valid, and the Fourier transform[3] is necessary. The current source can be broken up into two parts, namely the modulated part and the mode lock part. This is described in Eq. 8:

$$I(t) = I_1(t) \times I_2(t), \qquad (8)$$

where

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[3]The Fourier transform pair used in this note is:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt \Leftrightarrow f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}\,d\omega.$$

$$I_1(t) = 1 + m\cos(\omega_s t) \Leftrightarrow I_1(\omega) = 2\pi\delta(\omega) + \pi\left(\delta(\omega-\omega_s) + \delta(\omega+\omega_s)\right), \tag{9}$$

and

$$I_2(t) = \sum_{q=-\infty}^{\infty}\sum_{n=0}^{p-1}\delta\left(t - nT_{RF} - qT_L\right) + r\delta\left(t - pT_{RF} - qT_L\right) \Leftrightarrow$$

$$I_2(\omega) = \sum_{q=-\infty}^{\infty}\sum_{n=0}^{p-1} e^{-i\omega\left(nT_{RF}+qT_L\right)} + re^{-i\omega\left(pT_{RF}+qT_L\right)}. \tag{10}$$

Since Eq. 8 is a multiplication in the time domain, then $I(\omega)$ can be found by the convolution of Eqs. 9 and 10:

$$I(\omega) = \frac{1}{2\omega} I_1(\omega) \otimes I_2(\omega).$$

$$= \frac{1}{2\pi}\left\{2\pi\delta(\omega) + \pi m\left(\delta(\omega-\omega_s) + \delta(\omega+\omega_s)\right)\right\} \otimes \sum_{q=-\infty}^{\infty}\sum_{n=0}^{p-1} e^{-i\omega\left(nT_{RF}+qT_L\right)} + re^{-i\omega\left(pT_{RF}+qT_L\right)}.$$

Rewriting this slightly results in:

$$= I_{series}(\omega) + I_{source}(\omega). \tag{11}$$

The first term in Eq. 11 is simply the identical result as was obtained from the Fourier series analysis. This is simply

$$I_{series}(\omega) = \delta(\omega) \otimes \sum_{q=-\infty}^{\infty}\sum_{n=0}^{p-1} e^{-i\omega\left(nT_{RF}+qT_L\right)} + re^{-i\omega\left(pT_{RF}+qT_L\right)}$$

which equals

$$I_{series}(\omega) = \sum_{q=-\infty}^{\infty}\sum_{n=0}^{p-1} e^{-i\omega\left(nT_{RF}+qT_L\right)} + re^{-i\omega\left(pT_{RF}+qT_L\right)}. \tag{12}$$

The second term in Eq. 11 is the modulation term from the source.

$$I_{source}(\omega) = \frac{m}{2}\sum_{q=-\infty}^{\infty}\sum_{n=0}^{p-1}\left(e^{-i(\omega-\omega_s)\left(nT_{RF}+qT_L\right)} + e^{-i(\omega+\omega_s)\left(nT_{RF}+qT_L\right)}\right) +$$

$$+r\left(e^{-i(\omega-\omega_s)(pT_{RF}+qT_L)}+e^{-i(\omega+\omega_s)(pT_{RF}+qT_L)}\right). \tag{13}$$

Equations 12 and 13 imply that additional sidebands are produced from the modulation of the source. For the direct beam harmonics (i.e. n x 402.5 MHz), these sidebands are $\frac{m}{2}$ in magnitude lower than the main carrier signal, and are located on both sides of the main carrier at $402.5\ MHz \pm f_s.$. For the laser harmonics (i.e. n x 201.25 MHz), the sidebands are also $\frac{m}{2}$ lower than the carrier of the sideband and are likewise located at $201.25\ MHz \pm f_s$.

The salient point of the modulated source analysis is that an additional complication comes from the modulation. While the sidebands on the signal for detection of the electron stripping from the laser is on the order of -20 dB lower than the carrier.

V. Conclusion

The mode locked laser has been discussed as a tool for measuring the transverse profile of the beam. The analysis has been presented and results shown. The signal detection and measurement of the signals are optimistic since the signal levels are so low. Making differential measurements (i.e. with/without the laser) offer the best chance to see these signals to offer a bit more dynamic range, but this raises important issues not limited to: pickup accuracy, differential accuracy, amplifier control, equalizer fabrication. One piece of information not analyzed in this paper is a source that is switched on and off (at 60 Hz) as the SNS source would be. This will reduce the higher frequency harmonics and will make the higher order harmonics even more difficult to detect. A thorough analysis of other laser techniques is necessary before proceeding with this technology.

Acknowledgements: I wish to thank Dr. Saeed Assadi, Dr. A. Aleksandrov, Dr. W. Grice, and T. Shea for our discussions.