Effect of Transverse Image Charge Field on SNS MEBT Chopper Performance.

SNS/AP TECHNICAL NOTE

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It was pointed out [1] that the force from the image charge in the deflecting plates could affect the SNS MEBT chopper performance. The space charge electric field distribution deviates from the free space solution in presence of conducting surfaces close to the beam. The field from a set of image charges can represent this deviation. Considering chopper plates as infinite flat conducting surfaces and the beam as a line charge (fig.1), the electric field from the image can be easily found [2]:

$$E_y^{image} = \frac{p\ l}{48e_0h^2}(y+2d)$$

where $l = \frac{I}{bc}$ is linear charge density, h is half width of the gap, and d is beam deviation from the symmetry plane. At the center of the displaced beam $(y = d)$,

$$E_{y0}^{image} = \frac{p\ l\,d}{16e_0h^2}\ .$$

In the first approximation, deviation of the beam from the symmetry plane due to the main deflecting field is

$$d = \frac{eE_0}{2b^2mc^2}z^2 \quad ,$$

where E_0 is the chopper field, $b = \frac{v}{c}$, and z is the coordinate along deflecting plate. Then integrated field from the image charge is then

$$E_{y\,\mathrm{int}}^{image} = \int_0^L \frac{p\ l}{16e_0h^2}\cdot\frac{eE_0}{2b^2mc^2}\cdot z^2dz = \frac{p\ l}{96e_0}\frac{eE_0}{b^2mc^2}\frac{L^3}{h^2}$$

In order to estimate the perturbation of the deflection caused by the image charge, we should compare this field with the integrated main deflecting field $E_0 \cdot L$. Their ratio $\boldsymbol{d}$ is:

$$\boldsymbol{d} = \frac{E_{y\,\mathrm{int}}^{image}}{E_0L} = \frac{p\,e}{96e_0b^3mc^3}\frac{I\cdot L^2}{h^2} \approx 0.0276\cdot\frac{I[mA]L^2[m^2]}{h^2[mm^2]}$$

For SNS parameters of $I \approx 300mA,\ L = .3m,\ h = 9mm, d \approx 1\%$.

We should also consider focusing action of image charge field. A gradient of E_y^{image} provides a focusing force equivalent to the action of magnetic quadrupole with gradient $G_y^{eq} = \frac{\nabla_y E_y^{image}}{\beta c}$. Integrated the strength of an equivalent quad gives

$$K_y^{eq} = G_y^{eq} \frac{L}{3} = \frac{\pi IL}{144\varepsilon_0 h^2 \beta^2 c^2} \approx 45[Gs] \cdot \frac{I[mA]L[m]}{h^2[mm^2]}$$

For SNS parameters of $I = 300mA, \; L = .3m, \quad h = 9mm, \quad K_y^{eq} = 50Gs$. It should be compared with the focusing strength of adjacent quad of $1.12 \cdot 10^4 Gs$. The relative focusing error due to the image charge is only about 0.45%.

In the simplified analysis above we represent the beam by a line charge. This approximation is good only when transverse beam size is much less than the gap between conducting plates. In order to check the validity of this approximation for SNS parameters, the Poisson equation was solved numerically for a gaussian charge distribution between parallel conducting plates. The calculated dependence of the vertical component of the image charge induced electric field vs. the vertical coordinate is shown in figure 2 by the red line. The blue line shows the approximate solution. The average calculated deflecting force in the beam center location is very close to approximate solution. The calculated focusing strength is about twice that predicted by the approximate solution, but it is still less than 1% of the nearest quad force and can be neglected. The magnitude of the field non-linearity is a fraction of the focusing strength and also can be neglected. Note that this calculation was done for a maximum beam deviation of 4.5mm at the exit of deflecting plates. The chopper target will absorb a beam with this level of displacement. A partially chopped beam has less than half the deviation from axis, and the field induced by the image charge is proportionally smaller. We can conclude that the effect of the image charge transverse field on the chopper performance is negligible for SNS MEBT parameters.

References:

[1] G. Rees. Private communication.
[2] M. Reiser, Theory and design of charged particle beams.

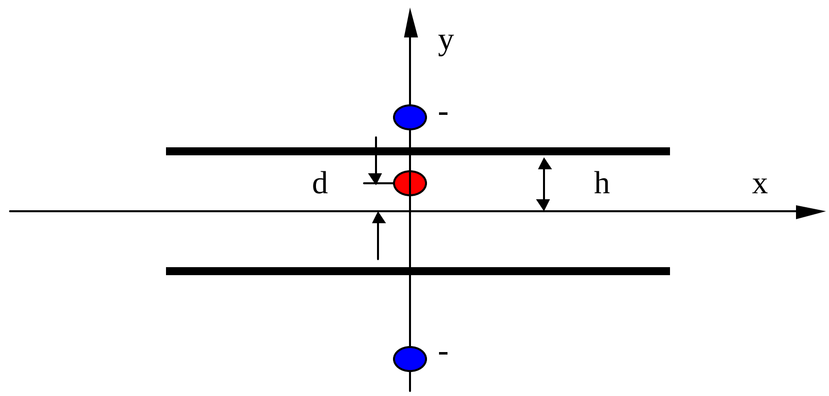

Figure 1. Geometry of the problem.

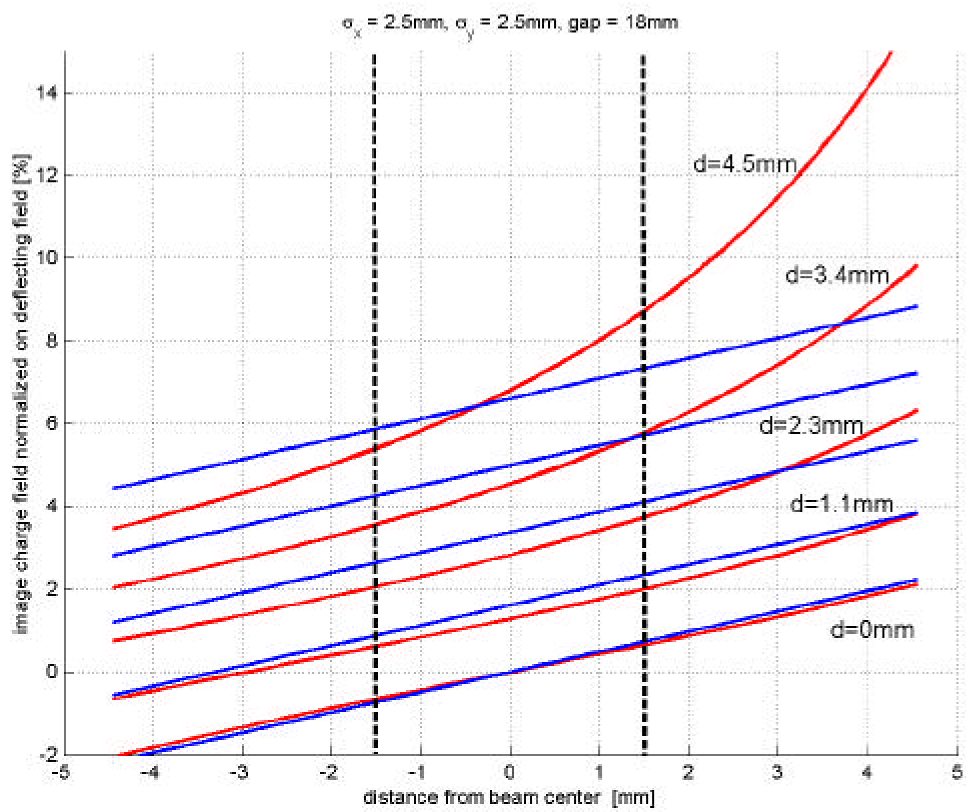

Figure 2. Dependence of the image charge field upon coordinate.