ORNL Accelerator Physics Memo

PROPOSAL FOR SOLENOID LENS EXPERIMENT AT THE PSR

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3. Motivation

Prior to the recent upgrade program, the average beam current in the PSR was limited to 70μA, at 20Hz and $2x10^{13}$ protons/pulse. This limit is the result of two separate problems. The first problem is uncontrolled beam loss and the second problem is the so-called "PSR e-p Instability". At 70μA and 20Hz the PSR uncontrolled beam loss is about 0.5μA, which severely activates the ring. This beam loss is believed to result from: (1) the production, at injection, of excited H^0 states that Stark strip in subsequent magnetic fields; and (2) nuclear and large angle Coulomb scattering resulting from an average of 300 foil hits per proton. These two mechanisms are estimated to contribute equally to the losses, and are correctable in a new spallation neutron source by changing the injection scheme.

Also at the $2x10^{13}$ protons per pulse level, the threshold for the PSR e-p Instability appears. The PSR e-p Instability limits the performance of the LANSCE facility and provides a fundamental uncertainty and risk to the performance of a new spallation neutron source. Although this instability has been studied for more than a decade by many capable people, there is not a consensus among accelerator physicists regarding its cause. Present indications are that this instability remains, despite the replacement of a two-step H^0 injection system by an improved one-step direct H^- injection system as part of the PSR upgrade program. Because of this instability the SNS vacuum system, clearing electrodes, diagnostics, bunching system voltage, foil injection system, and damping system are sources of considerable concern. These systems may be over- or under-designed depending on the physics mechanism(s) of the PSR e-p Instability, which are unknown at present.

There are several possible explanations of the PSR Instability. It has been found[1] that increasing the betatron tune spread with octupoles increases the instability threshold. In particular, increasing the betatron tune spread by about 0.01 leads to an increase in the threshold by about a factor of three[1]. Unfortunately, octupoles do not provide a practical means of overcoming the PSR Instability. Although the octupoles do increase the instability threshold, they also increase the uncontrolled beam loss by exciting nonlinear resonances. Even so, this is an important result because it shows that modifying the magnetic fields to increase the tune spread can effect the instability threshold. Additional discussion of this phenomenon can be found in [1]. Increasing the energy spread can also increase the tune spread in the PSR; however, this is not a viable solution because of the limited horizontal aperture.

In the theoretical study of dynamic systems, the field of integrable optics seeks mathematical conditions that, if satisfied by the Hamiltonian, guarantee the existence of constants of the motion. For such cases the motion will be nonchaotic, and perhaps even nonresonant. In its application to accelerator beam dynamics, integrable optics seeks to modify the Hamiltonian, rendering the motion regular, through the use of magnetic lenses. Such an approach may be useful for stabilizing the PSR instability by increasing the tune spread, similar to the octupole, without introducing resonances.

The integrability conditions are restrictive, which tends to limit their applicability to accelerators, but there are a few ready-to-use examples. One of those examples is related to round beam conditions. The system requires at least one thin, in comparison with the beta functions, lens at a point with equal horizontal and vertical beta-functions; the working point is near a difference resonance with the fractional part of the tunes equal to 0.25 or 0.75. Since a nonlinear lens has its own linear focusing, the final working point is assumed to be about 0.35- 0.37. The maximum tune spread in this case is determined by the maximum available gap between the half-integer resonance and the working point, and is about 0.1. These conditions are very close to the operational regimes of the spallation neutron sources (PSR in Los Alamos and SNS in Oak Ridge).

We propose to install a solenoid lens in the PSR ring. This will increase the tune spread of the beam, in effect providing Landau damping similar to octupoles, but without exciting nonlinear resonances and without increasing the energy spread of the beam. A similar idea, also motivated by integrable optics, has been used with some success to counteract the nonlinear motion from collider beam-beam effects in the existing CESAR ring at Cornell (Ref. 2 and references therein). If our proposed experiment is successful, the PSR Instability will be averted and a similar lens can be installed in the SNS Accumulator ring.

4. Axially Symmetric Lens for Integrable Optics

This is a proposal to increase the threshold of the PSR e-p instability, without producing betatron resonances or energy spread, by increasing the Landau damping and betatron tune spread using a solenoid lens. This proposal rests upon the concept of integrable optics, which is a branch of Hamiltonian dynamics that determines how to modify dynamic systems to obtain integrable Hamiltonians having regular phase space maps and no chaos. For such cases, the particle trajectories, when integrated as functions of time and initial coordinates and momentum, follow the invariant surfaces. In essence, the maps are similar to linear maps with the addition of a betatron tune spread, but the magnetic fields required to obtain such maps are complicated.

One particularly simple class of integrable optics applies to axially symmetric magnetic fields. Under certain conditions, solenoid lenses can be used to construct integrable maps for circular rings. To apply this to PSR, the lattice must meet two conditions: (1) the solenoid lenses must be installed in a straight section at a location with $\beta_x = \beta_y$, so that a beam with equal emittances is round; and (2) the fractional tunes of the ring in both planes, without the solenoids, must be either 0.25 or 0.75. Once these conditions are satisfied, one integral of the motion, the angular momentum, is provided by the axial symmetry of the system:

$$M = x \cdot p_y - y \cdot p_x\,, \tag{1}$$

where x, p_x are the horizontal, and y, p_y are the vertical coordinates and momenta, respectively. M is conserved in the linear storage ring structure.

There is an additional invariant of motion, which is conserved for all values of the angular momentum whenever the nonlinear kick is axially symmetric and the change in radial momentum p_r is given by:

$$\Delta p_r = -\frac{b \cdot r}{(1 + a \cdot r^2)}\,, \tag{2}$$

where a and b are free parameters (Ref. [3] and references therein). Throughout this proposal we use the normalized variables, $x_n = \frac{x}{\sqrt{b_x}}, y_n = \frac{y}{\sqrt{b_y}}$, and their derivatives with respect to the betatron phase, $x'_n = \frac{b_x \cdot x' - \frac{b'_x \cdot x}{2}}{\sqrt{b_x}}, y'_n = \frac{b_y \cdot y' - \frac{b'_y \cdot y}{2}}{\sqrt{b_y}}$, where primes denote derivatives with respect to longitudinal coordinate in R.H.S. and the subscript n is suppressed. In particular, the momentum, p_r, is the derivative of normalized radius, $r_n = \frac{\sqrt{x^2 + y^2}}{\sqrt{b}}$, with respect to the betatron phase (where $b_x = b_y = b$). If the linear transfer matrices for one turn around the ring for both horizontal and vertical normalized variables are given by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, then, the invariant after the above lens is:

$$\mathrm{I}(r_n, p_r) = (a \cdot r_n{}^2 + 1) \cdot (r_n + p_r)^2 + b \cdot r_n \cdot (r_n + p_r) + (r_n{}^2 + \frac{M^2}{r_n{}^2})\,. \tag{3}$$

Figure 1 plots the invariant surfaces given by Equation (3). In order to model PSR, we scale the x coordinate and dynamic aperture close to that of PSR (5 cm); the total scale is 10 cm (1 dm). For this calculation the angular momentum is equal to zero and the coefficients are a = -0.40 (in the units of $scale$; the scale for a is $\frac{b}{dm^2}$) and b = 1.17, so the reference orbit betatron fractional tune for both directions is 0.35 with the solenoid present.

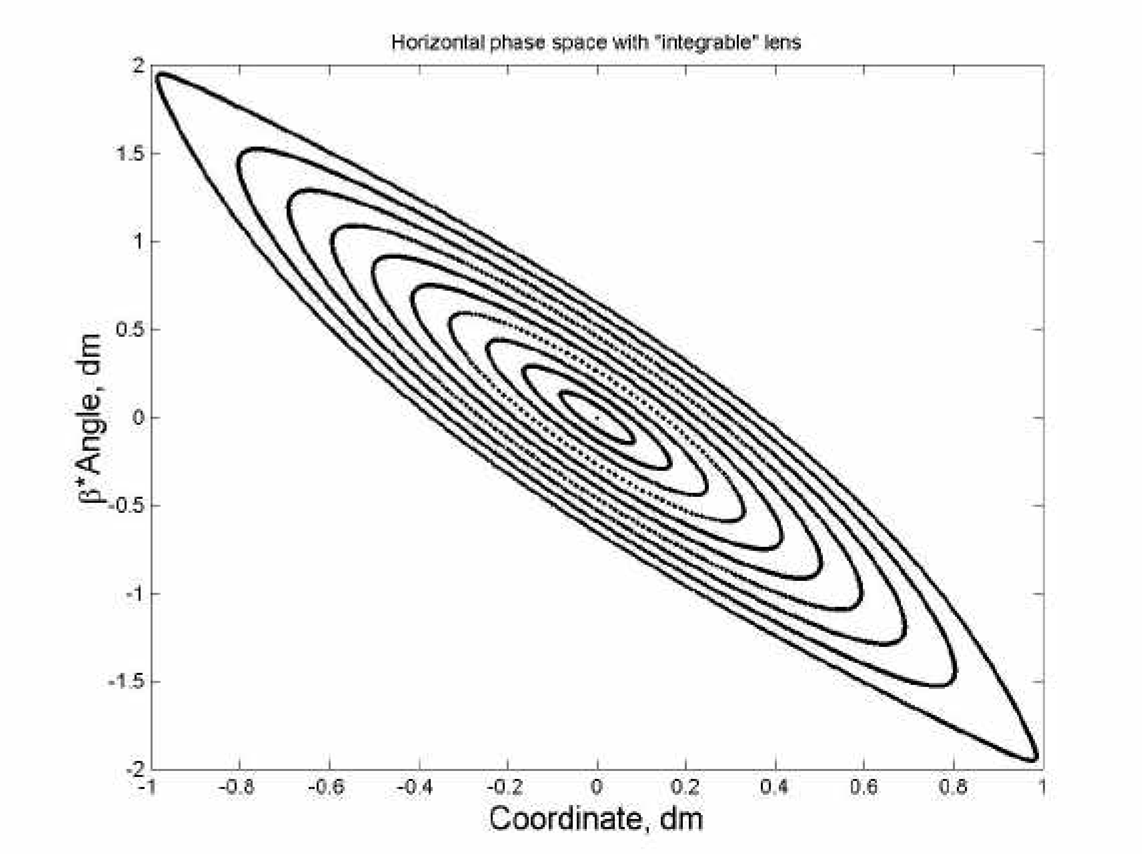

Fig. 1. Phase space for the motion in the integrable cell. The ten contours represent the motion with varying initial conditions, from the closed orbit up to the limit of the dynamic aperture.

The stable motion is bounded by two separatrices, which occur in normalized variables at the points:

$$-\sqrt{{(b-2)}/{2a}},-2\sqrt{{(b-2)}/{2a}} \text{ and } \sqrt{{(b-2)}/{2a}},2\sqrt{{(b-2)}/{2a}}. \qquad (4)$$

These two points are critical points of the Equation (3) invariant and define the region of stable phase space. For the above parameters, the normalized coordinates and momenta of these stationary unstable points are -1.02, -2.04 and 1.02, 2.04. These dimensionless values can simplify tracking, but the un-normalized expressions for the needed parameters can also be obtained directly.

An upper limit to the total spread in tunes for this case is 0.5 – 0.35 = 0.15 (0.5 corresponds to the half-integer resonance having two saddle points and the separatrix as the stable motion boundary, while 0.35 is the closed orbit frequency), if the beam boundary is at the dynamic aperture. This is not a realistic situation – usually the average beam size is several times smaller than dynamic aperture. If, for example, the beam occupies half the aperture, then the tune spread (found numerically) is 0.006 and is close to what we need. This increase in tune spread is produced without exciting any betatron resonances. The detailed formula for the spread is presented below in the section that deals with the real parameters of the lens and the PSR parameters.

We must form the lenses out of an axially symmetric magnetic structure in order to preserve angular momentum invariance. All the fields in an axially symmetric system can be calculated from the single function H(s), the longitudinal magnetic field on the axis of the lens, which corresponds to the reference orbit. The longitudinal magnetic field in all space is then given by (Ref. [4]):

$$H(s,r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \cdot H^{(2n)}(s) \cdot (r/2)^{2n}, \tag{5}$$

where r is the radius from the axis, s is the longitudinal coordinate, and the superscript (2n) denotes the 2n-th derivative of the longitudinal magnetic field on the axis H(s) with respect to the coordinate s. Calculation of nonlinear kicks from such a lens up to third order in coordinate was made, see for example Ref. [5]. Here we will write down the exact expressions for the nonlinear kick.

The radial and azimuthal motion of a particle can be described by the two simple equations:

$$r'' - r\,q'(\,q' + \frac{e}{p} \cdot H_s) = 0, \quad q' = q_0' \cdot r_0^2 / r^2 - \frac{e}{2p} \cdot \frac{\Phi}{pr^2}, \tag{6}$$

where the derivatives are taken on the longitudinal coordinate, q, q_0 are the azimuthal and initial azimuthal angle of the particle, and the same notation is used for the radius r. The symbol Φ stands for the magnetic flux and is equal to $\Phi = 2p\int_0^r H_s \cdot r'dr'$, where H_s is taken from Equation (5). Finally, the equations in (6) can be combined to yield:

$$\begin{aligned} r'' - r \cdot (q_0 \cdot {r_0}^2/r^2)^2 = \frac{e}{p} \cdot q_0 \cdot {r_0}^2/r^2 \left(\frac{\Phi(r)}{pr^2} - H_s(r)\right) + \\ \frac{e^2}{(2pp)^2 r^3} \cdot (\Phi(r)^2 - \Phi(r) \cdot \Phi(r)'_r \cdot r). \end{aligned} \tag{7}$$

The force term on the L.H.S. of Eq. (7) is just the centripetal force, which always appears in cylindrical coordinates. The first term on the R.H.S. has no influence for the short lens. It is a linear function of the longitudinal field derivatives at the axis of the system, and its integral is zero because the function at the ends is zero. Consequently, only the last term of the R.H.S. is of interest, and is denoted by:

$$\begin{aligned} K(r,s) = \frac{e^2}{p^2}(-\frac{1}{4}(H^{(0)})^2 \cdot r + \frac{1}{8} \cdot H^{(0)} \cdot H^{(2)} \cdot r^3 - (\frac{1}{128} \cdot H^{(0)} \cdot H^{(4)} + \frac{3}{256} \cdot (H^{(2)})^2) \cdot r^5 + \\ (\frac{1}{4608} \cdot H^{(0)} \cdot H^{(6)} + \frac{1}{768} \cdot H^{(2)} \cdot H^{(4)}) \cdot r^7 - \ldots) \end{aligned} \tag{8}$$

where $H^{(n)}$ is the n-th derivative of the longitudinal field on the axis.

Let us assume that the coordinate and beta functions of the system change only slightly over the length of the lens of our axially symmetric system. The radial kick can be obtained by integrating $K(r,s)$ over s, regarding the radius r as a constant. The integration yields, where we ignore the centripetal force and consider only the kick from the magnetic field:

$$\begin{aligned}\Delta r' = \frac{e^2}{p^2}\Big(&-\frac{r}{4}\int_{-\infty}^{\infty}(H^{(0)})^2 ds - \frac{r^3}{8}\int_{-\infty}^{\infty}(H^{(1)})^2 ds \\ &- \frac{5\cdot r^5}{256}\int_{-\infty}^{\infty}(H^{(2)})^2 ds - \frac{7\cdot r^7}{4608}\int_{-\infty}^{\infty}(H^{(3)})^2 ds - \ldots\Big)\end{aligned} \tag{9}$$

This equation is for un-normalized variables, the coordinate r and its longitudinal derivative r'. This kick may be written in terms of the normalized variable $r_n = r/\sqrt{b}$ and its derivative r_n' over betatron phase $y = \int_0^s \frac{ds'}{b}$ as:

$$\begin{aligned}\Delta r_n' = \frac{e^2}{p^2}\Big(&-\frac{r_n\cdot b}{4}\int_{-\infty}^{\infty}(H^{(0)})^2 ds - \frac{r_n^{\,3}\cdot b^2}{8}\int_{-\infty}^{\infty}(H^{(1)})^2 ds \\ &- \frac{5\cdot r_n^{\,5}\cdot b^3}{256}\int_{-\infty}^{\infty}(H^{(2)})^2 ds - \frac{7\cdot r_n^{\,7}\cdot b^4}{4608}\int_{-\infty}^{\infty}(H^{(3)})^2 ds - \ldots\Big)\end{aligned} \tag{10}$$

In summary, we propose to construct an axially symmetric lens from solenoids. The magnetic fields from the lens can be expressed in the form of Eq. (5). The on-axis derivatives of this field can then be used to calculate the momentum kicks from this lens as a function of radius. These kicks must have the same radial dependence as Eq. (2). The parameters a and b in Eq. (2) are chosen to meet the following conditions:

1. Physically realizable with solenoid lens;
2. Yield separatrices, in Eqs. (3,4), that span the transported phase space;
3. Provide the largest possible tune spread;
4. Provide a minimum overall linear tune shift for the lattice that is equal to $\frac{b}{4p}$;
5. Minimize the magnet current.

5. Particular Requirements for the PSR

The PSR lattice is not complicated and consists of ten FODO cells with each FODO cell having a missing magnet. The quads are divided into two power-supply families, focussing and defocusing consequently the beta functions and tunes are highly coupled. The solenoid lens will provide a linear tune increase of about 0.1 in both planes, consequently the two quad power supplies will be adjusted to provide tunes, without the solenoids, of ν_x =3.25 and ν_y =2.25, so that the tunes with the solenoids will be ν_x =3.35 and ν_y =2.35, as required for the integrable optics. With these tunes the solenoids will be placed at a location where $\beta_x = \beta_y$. We calculate the beta functions to be $\beta_x = \beta_y$ = 7.0m at this location.

The acceptance of the PSR is 120 πmm mrad. At the location of the solenoid we will need clear space for a beam radius of 35mm allowing 5mm for closed orbit error.

6. Physics Design of Axially Symmetric Lens from Solenoids

Now let us construct the integrable lens from one-wire coils. The magnetic field from a one-wire coil on the symmetry axis is:

$$H(T) = 2p\ 10^{-7}\, I(A) \cdot \frac{R^2}{(R^2+s^2)^{3/2}}, \tag{11}$$

where *R* is the radius of the coil, *I* is the current in Amperes, and *s* is the longitudinal coordinate with respect to the coil. It has found numerically that two coils with the same longitudinal coordinate can be utilized to produce the needed nonlinear lens with very good accuracy. A ninth order Taylor expansion of the kick on the radial coordinate found relative coil radii of *R* and *1.5R*, and relative currents of *I* and *–0.65I*, respectively. Figure 2 presents a schematic view of this lens.

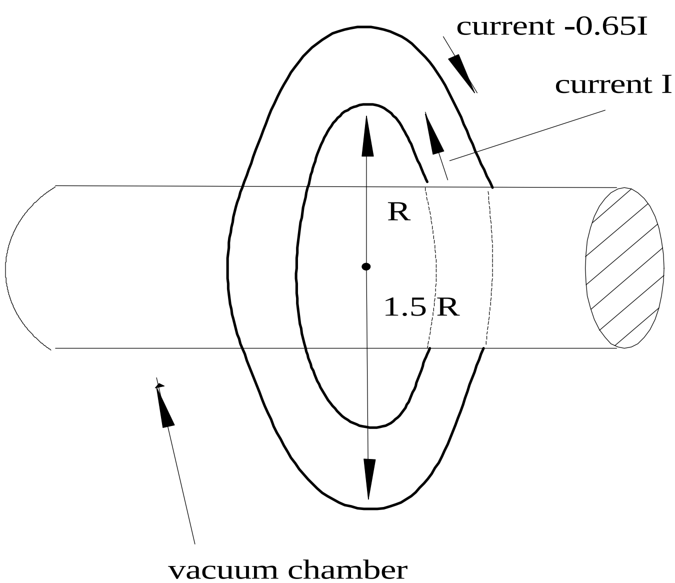

Figure 2 Schematic view of nonlinear lens

Now let us write the parameters of this lens in terms of the inner radius *R* and current *I*. The radial angle change is:

$$\Delta r' = \frac{R^2}{r_L^{\,2}}\left(-0.073\frac{r}{R} - 0.064\frac{r^3}{R^3} - 0.6\frac{r^5}{R^5} - 0.055\frac{r^7}{R^7} - 0.049\frac{r^9}{R^9}\ldots\right), \tag{12}$$

where $r_L(m) = \frac{\frac{v}{c} E(GeV) \cdot R(m)}{0.3 \cdot 2p \cdot 10^{-7} \cdot I(A)}$ is the Larmour radius corresponding to the maximum magnetic field at the axis. The ratio of all successive pairs of coefficients is nearly constant, and is equal to 0.9 ± 0.02. This provides the value of a for Equation (4): $a = -0.9 \cdot \frac{b}{R^2}$. The coefficient $b = 1.17$ for the same equation is $\frac{0.073 \cdot R \cdot b}{r_L^2}$. The dynamic aperture is $\sqrt{(b-2)/2a}$, where this expression stands for normalized coordinate. In usual coordinates it is $D = \sqrt{(b-2)/2a} \cdot \sqrt{b}$.

To obtain a linear shift from this lens the coefficient b (Figure 1) must be 1.17. Substituting the parameter values $R = 0.05m,\ b = 10m$ appropriate to PSR, we obtain $r_L = 0.2m,\ D \approx 5cm$ for Larmour radius and dynamic aperture, respectively, and the coil current is: $I(A) = \frac{\frac{v}{c} \cdot E(GeV) \cdot R(m)}{0.3 \cdot 2p \cdot 10^{-7} \cdot r_L(m)} \approx 2 \cdot 10^6 A$.

It is important to emphasize that two lenses are needed to compensate the coupling from the solenoids. In this particular variant they must be placed far enough apart to prevent overlap of their magnetic fields in order to keep the above calculations accurate.

7. Tracking Studies with Axially Symmetric Solendoid Lens Kicks

The particle tracking code SAMBA was used to simulate the affect of a solenoid lens for PSR. The nominal PSR lattice was used, and a thin lens kick was inserted in the middle of section 2.1, at a distance 16.68 m downstream from the foil. The defocusing quad strengths of the magnets QU1 and QU2 were increased from k=-0.46215m^2 to k=-0.48647 m^2. The focusing quad strengths of magnets QFS1 And QFS2 were increased from k= 0.7541 m^2 to k=0.8379 m^2. With these quad strength adjustments, the lattice tunes are $\nu_x = 3.25$ and $\nu_y = 2.25$. Also, the beta values at the proposed solenoid location are similar: $\beta_x = 6.84$ and $\beta_y = 6.97$. So the necessary criteria for the integrable solenoid invariant are satisfied. Macro-particles were sampled from truncated Gaussian distributions at the foil and tracked for 100 turns. Particle distributions were picked with similar RMS emittances as those arrived at with typical PSR painting schemes, and were truncated at 60 π-mm-mrad horizontal and 50 π-mm-mrad vertical. The thin lens kick is given as in Equation 2. Typically, 2000 macro-particles were tracked for each case presented for 100 turns.

The motivation of this exercise is to find a combination of the linear and non-linear kick components (i.e. coefficients *a* and *b* in Eq. 2) that: (1) provided as large a tune spread as possible and (2) minimize the growth in the beam size. The latter point is important because even though the particle orbits with the solenoid lens kick are stable, they may possibly be larger than before, since even a linear lens produces beta functions deviations and a beam size increase. We wish to find a system that perturbs the beam size as little as possible. To examine the beam size growth with the solenoid lens strength, we monitor the fraction of the beam that has grown to an emittance of 70 π-mm-mrad in either direction. This is a modest increase over the original beam emittance mrad and is comparable to what may be expected from space charge growth anyway.

Figure 7.1 shows the impact on the attainable tune spread for various combinations of the linear and nonlinear coefficients *b* and *a*. Figure 7.2 shows the corresponding percentage of the beam which has grown to 70 π-mm-mrad. The attainable tune spread increases with both *a* and *b*, as does the beam size. In order to attain a spread of ~ 0.01, with less than 1% of the particles growing to emittances > 70 π-mm-mrad, it is desirable to have a small linear coefficient (< 0.04 mm^{-1}) and a large nonlinear coefficient (~ $8x10^{-4}$ mm^{-2}). These values correspond to having a small contribution from the first term on the RHS of equation 9, and larger contributions from the higher order terms. In the following section we investigate the feasibility of attaining these integrated field contributions from various solenoid coil configurations.

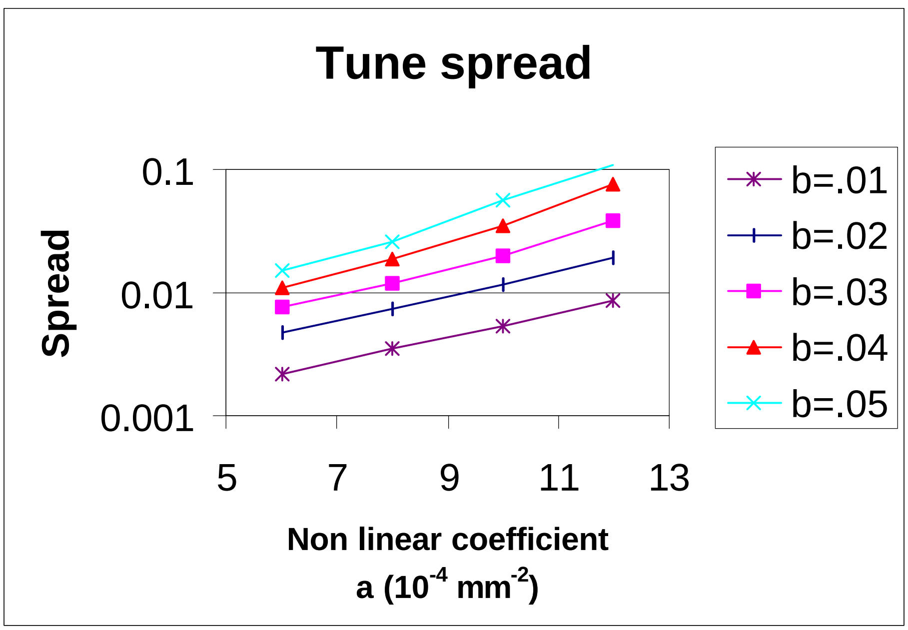

Figure 7.1 Tune spread for different combinations of linear (*b* in mm^{-1}) and nonlinear (*a* in 10^{-4} mm^{-2}) kick coefficients.

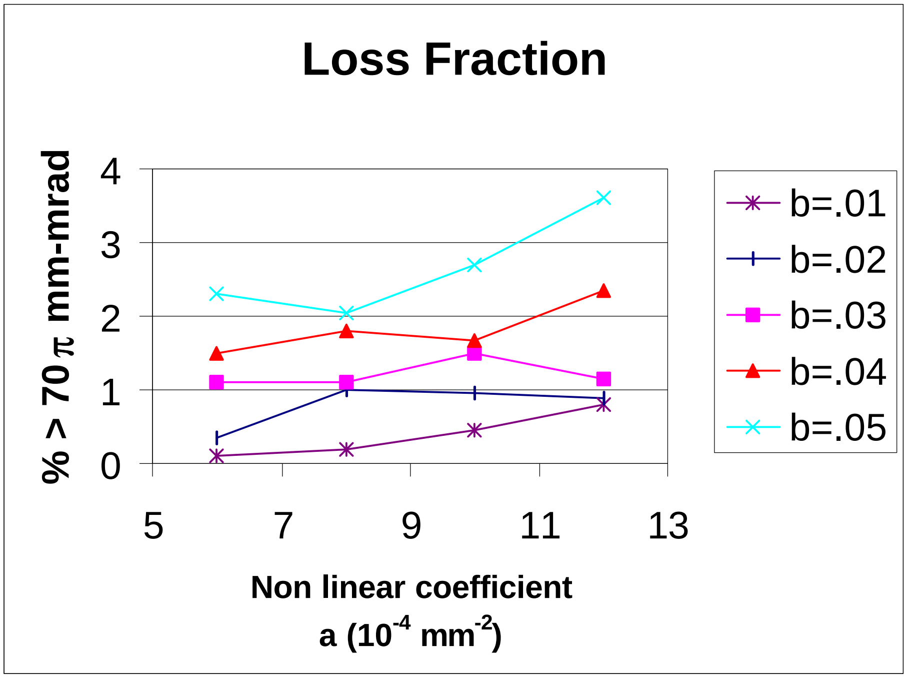

Figure 7.2 Fraction of the beam that has grown to 70 π-mm-mrad after 100 turns for different combinations of linear (*b* in mm^{-1}) and nonlinear (*a* in 10^{-4} mm^{-2}) kick coefficients.

8. Mechanical Design of the Axially Symmetric Lens

Steady-state Copper Coils

Another solution could be implemented using wound copper wires with water cooling. We consider a set of six solenoids with alternating +,- currents as shown in figure 8.1. The minimum radius of the coils, r_{min}, is taken to be 3.5 cm, as discussed above. Each coil is constructed by winding a 2kA cable with a 6.35 x 6.35 mm cross section (1/4 inch), corresponding to a current density of 5 kA/cm^2. These parameters are related to the available power supply (2 kA) and the minimal available cross section wire with water cooling. We investigate the feasibility of creating large tune spreads for various current/coil, coil cross section aspect ratio A_{coil} = W/H and coil separation *ds*. Each coil is divided into I(kA)/2 filaments, which are distributed according to the coil aspect ratio. The field and its gradients are calculated along the symmetry axis by summing to components from each filament (using eq. 11 and its gradients).

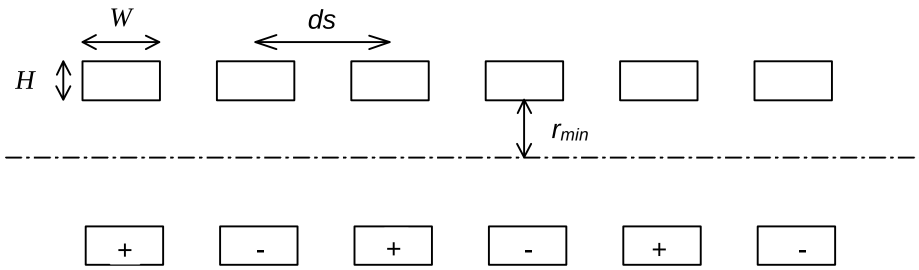

Figure 8.1 Schematic layout of a 6 coil solenoidal lens.

The most important for us is the second integral on the RHS of Eq. 9, since it determines the cubic nonlinearity and betatron tune spread for small and medium amplitudes where the most part of the beam is initially. The betatron tune spread could be easily expressed via this integral:

$$\Delta\nu = \frac{3\beta^2\varepsilon_0}{128\pi}\left(\frac{0.3}{E(Gev)v/c}\right)^2 \int_{-\infty}^{\infty} (H')^2\, ds \;, \qquad (13)$$

where all other parameters were defined above and the integral should be taken in Tesla^2/m.

Figure 8.2 shows the value of the second integral on the RHS of Eq. 9 for various total currents and geometries (units are T^2/m , normalized by $(\mu_o\ I/2\ R)^2$ /R with I written in the title of each figure and R is 3.5 cm). This figure shows the efficiency of each particular configuration of the coils. The coil spacing parameter *ds* in these figures is normalized by R.

Figure 8.2 shows that the cubic nonlinearity integral goes down as the current and, correspondently, the coil thickness increase. The additional space requirements for higher currents causes the coils to cancel each other. For higher current levels, larger spaces between the coils help, but there is still a net lowering of the higher order integrated $(dB/ds)^2$.

Let’s calculate the betatron tune spread for the 400 kA each coil current for *ds* is equal to 4 R and the aspect ratio equal to 1.4 (it roughly corresponds to the maximum of the cubic nonlinearity). After substituting all the parameters in Eq. (13) we have it equal to $0.3*10^{-3}$
In order to provide a tune spread of ~ 0.006, we need about 20 of those solenoids. While each of them has the length of about 60cm, total length would be 12 meters and total power loss would be huge. In order to increase this integral, iron can be included, higher current density superconducting coils can be used, or higher current density pulsed coils can be used.

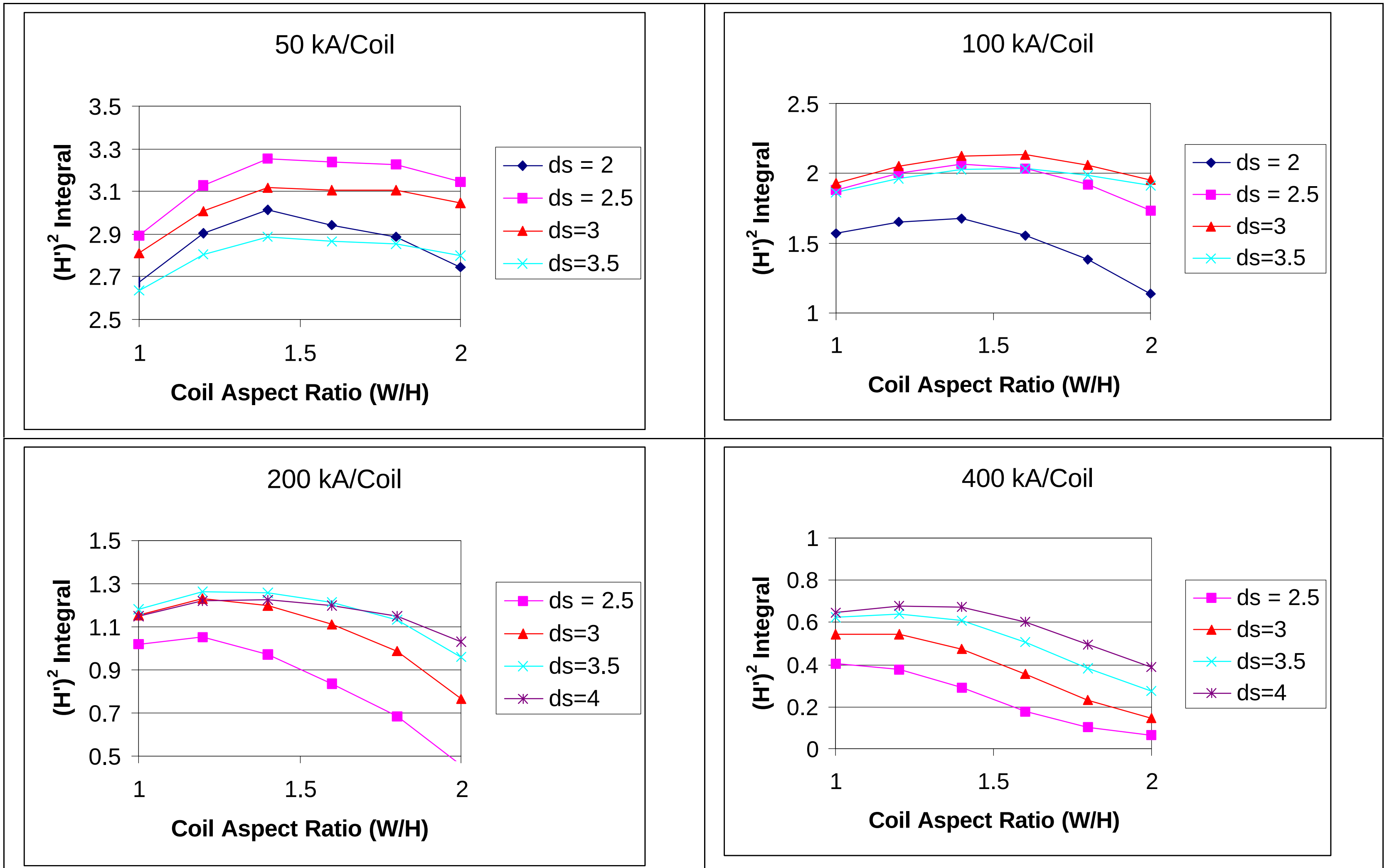

Figure 8.2 Magnitude of the (H') integral for various total currents/coil and geometries for the steady-state copper coil option.

Superconducting case

Now we want to present a superconducting variant and show that it looks more feasible. In the first variant we calculate the required current *I* for the inner coil to be about 2000 kA for one lens or 1400 kA for two lenses. The easiest way to create such a system is to use superconductors. The maximum current for 1 square millimeter of a superconducting cable is typically 200 A and 10 Tesla is the maximum field. Hence, the cross section of the inner coil must be about $10\times10\, cm^2$. The maximum field at the axis is about 10 Tesla. We can assume that the field at the coil will be almost the same, so it is close to the threshold. The betatron tune spread can be calculated as a function of the rms emittance retaining only the cubic nonlinearity of the lens. The formula is $\Delta n = \frac{3}{8\cdot 2\cdot p}\cdot b\cdot a\cdot e_0 = 6\cdot 10^{-3}$ for emittance $e_0 \approx 20\cdot 10^{-6} m * rad$. According to the paper (Ref. [1]) this spread is enough to increase the threshold by factor of 1.5-2. This result can be considered as a first approximation to real nonlinear integrable optics. The most important result is that the suggested solution for the lens appears to be achievable. But optimal inexpensive design may not be so simple, since the current density and the magnetic field are close to technical limits. Also the cross section in this variant is still too big in comparison with the vacuum chamber cross section. Thus we consider a 10 coil variant with optimal coil aspect ratio from previous section. Namely we use an aspect ratio of 1.2 and coil spacing parameter *ds* of 3 from the 200 kA variant in Fig. 8.2. However the current in the coil is taken to be 600 kA due to much higher available current density in the superconductor. Additionally, we consider inclusion iron surrounding the coils as in Figure 3. This provides a 65 % increase in the cubic non-linearity over the case with no iron. Figure 3 shows the end of the coil configuration used to model the impact of the iron inserts calculated with the Los Alamos code “Poisson”. Contours of constant field level are also shown.

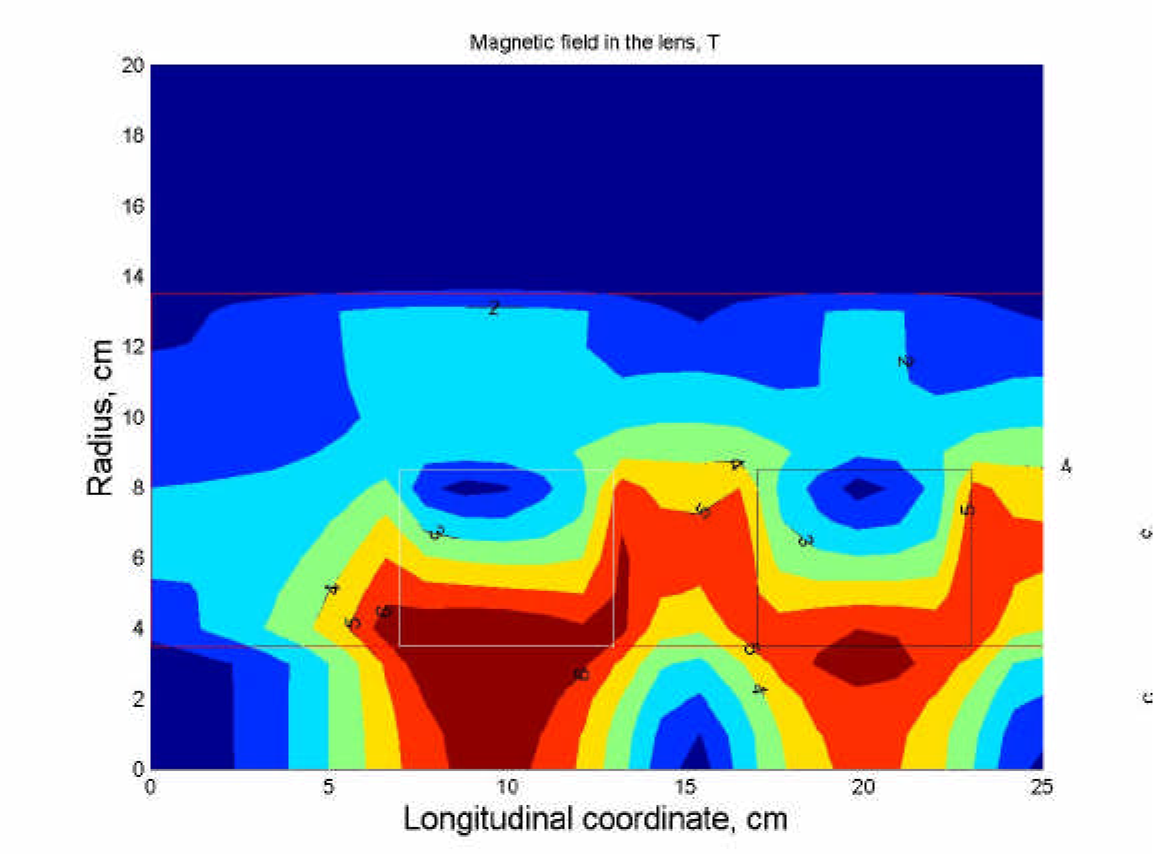

Figure 3 Magnetic field in superconducting solenoid

The red contour shows the iron boundary, the white contour delineates the positive current area(600 kA) and the black contours delineates the minus current (-600 kA) area. One can see that the maximum magnetic field is about 6 Tesla at the coil. This field level allows the possible use of NbTi cable at 4.2K with an overall current density 200 A per square millimeter (in practice, it could be even up to 500 A per square millimeter). The betatron tune spread for this case is .004 what is close to what we need. A more expensive superconducting variant could use lower temperatures and higher fields permit operation at up to 14 T, which would correspond to a tune spread four times higher.

Pulsed case

For an experimental test, the tune spread need only be applied occasionally (say at 1 to 0.1 Hz). This offers the possibility of using high field, low duty factor coils, for example obtaining even higher fields (up to 20 Tesla). This regime can save power and provide us with higher fields. Initial estimates for pulsed power supply components for the same configuration as the superconducting solenoid discussed above are considered. Let’s assume that we connect all the coils in series. The resistance for this case is about $1.5*10^{-5}$ Ohm and because of the alternating current in the solenoid configuration, the inductance is low, namely about 10^{-6} Henry. The requirement could be matched with 200 MegaCap™ capacitors with parameters – 15 Volts, 100 Farads recharge time 0.6 seconds and current about 10 kA (price per unit is 400$). These provide a total current of about 1 MA while in parallel and the voltage for 1 MA is about 15 Volts as the capacitors have.

????Is this what we mean: –

The capacitors provide a total current of > 1 MA, which is more than adequate, and are well matched with the voltage of the coil system.
?????

The resistance of our solenoid is bigger than the reactive resistance ($=\sqrt{\frac{L}{C}} = 610^{-6}$ Ohm) so there are no oscillations in the capacitor discharge process and the voltage is not distorted much due to the LCR circuit oscillations. We need also high current tiristors to provide this supply with a fast switch. A one millisecond pulse requires about 10^4 Joules. Even if the repetition rate is about 20 Hz, the average power is only about 200 kW. The total energy capability from the capacitors is $200*100*15^2/2 = 2*10^6$ Joules. So this is ten times higher than what we need for 1 second of recharge time. So everything looks feasible with usual components. Also an estimation for time of magnetic field penetration through vacuum chamber is ~ 10 microseconds, which means that we can install this pulsed lens outside the vacuum chamber.

Solenoid specs
- How many
- Ampere turns
- Turns
- Amperes
- Cooling
- Voltage
- Conductor cross section
- Forces

9. Error Studies with Axially symmetric Solenoid Lens

What happens if beta X not = beta Y?

What happens if nu does not equal 0.25?

What happens if there is a lot of other x-y coupling?

What happens if the solenoid is miss aligned?

10. Requirements

Power supply requirements
Cooling water requirements

First, we should mention that, in fact, an even number of lenses is needed to compensate coupling.

11. Responsibilities: Hardware, Software and Labor

Fabrication of solenoid and delivery to PSR
Requirements for the location of solenoid in the PSR lattice. Round beam betax=betay.
Knobs to make the beta functions equal and nux and nuy quarter integer.
Does the emittance in the two planes have to be the same also?
Fabrication of mount and installation in the PSR
Finding existing PS
Connecting power supply, water, interlocks, and controls
Supplying PSR beam time
Performing measurements with beam

12. The Experiment

13. References

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