PROCESSING OF THE SIGNAL FROM PSR STRIPLINE BPM

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Abstract

This is a note on the processing of the signals from PSR strip line BPM that obtained in experiments on e-p instability. Method of calculation of beam centroid displacement from the pickup signals is presented. Two-dimensional Fourier transformation of the signals in presence of the fast instability is analyzed.

I. Introduction

Precise measurement of beam centroid movement is very important for understanding the instability in the Proton Storage Ring (PSR). Fast strip line BPM (WM41 or CERN BPM) can provide this information if proper processing algorithm is used. In the previous work on this subject [1] vertical difference signal obtained by analog subtraction of signal from top and bottom strip lines of BPM was analyzed.

In this work we analyze signals from top and bottom strip lines digitized separately and make further processing in digital form to eliminate errors in frequency dependent analog hybrid. Strip line frequency response is approximated to second order that improves accuracy considerably. Suggested algorithm allows to obtain a snap-shot of transverse beam position on each turn, than beam centroid movement on successive turns can be developed in series of plane travelling waves in beam frame of reference thus providing very important information on instability development.

II. Coordinate reconstruction

Voltage induced on strip line by moving charge can be expressed in terms of scalar product of charge velocity and static electric field of strip with unit potential. Therefore beam moving with small angle along strip will excite it on the edges and net voltage across upstream port is [2]:

$$u(t) = I(t)F(x(t)) - I(t-t)F(x(t-t)), \qquad (1)$$

where $I(t)$ is beam current, $F(x)$ is function depending on strip line geometry , $x(t)$ is distance from beam centroid to strip line, t is signal delay in strip line.

This expression is well applicable to signal excited by proton beam in PSR but not to that excited by transversely moving electrons if they exist. We will discuss possible contribution from electrons elsewhere but neglect it in further consideration here.

Rewrite (1) in general form:

$$u(t) = U(t) - U(t-t)\,, \qquad (2)$$

develop in series second term

$$U(t-t\ )=U(t)-U'(t)t\ +\frac{1}{2}U''(t)t^{2}-\frac{1}{6}U'''(t)t^{3}+\ldots.,\tag{3}$$

substituting (3) to (2) we have

$$u(t)=U'(t)t\ -\frac{1}{2}U''(t)t^{2}+\frac{1}{6}U'''(t)t^{3}+\ldots...,\tag{4}$$

integrate both sides

$$\int u(t)dt=U(t)t\ -\frac{1}{2}U'(t)t^{2}+\frac{1}{6}U''(t)t^{3}+\ldots.,\tag{5}$$

and obtain expression for $U(t)$

$$U(t)=\frac{1}{t}\cdot\int u(t)dt+\frac{1}{2}U'(t)t\ -\frac{1}{6}U''(t)t^{2}-\ldots.,\tag{6}$$

if we neglect all terms but first in RHS we obtain first order approximation used in [1].

$$U(t)=\frac{1}{t}\int u(t)dt,\tag{7}$$

Small parameter here is wt where w is harmonic frequency. CERN BPM has t =1.125ns and observed instability has typical frequency $w\approx 2\pi\cdot 100$MHz, so $wt\ \approx .7$ is not very small and first approximation is insufficient. We can improve accuracy to next order by deriving $U'(t)$ from (4)

$$U'(t)=\frac{u(t)}{t}+\frac{1}{2}U''(t)t\ +\ldots.,\tag{8}$$

and substituting it to (6) we obtain second order approximation for $U(t)$

$$U(t)=\frac{1}{t}\int u(t)dt+\frac{1}{2}u(t)\,,\tag{9}$$

Note that by differentiating (4) and substituting $U^{(n)}$ to (6) we can increase step by step an order of approximation. In this case derivatives of measured signal $u(t)$ will appear in RHS of (9), but digital differentiation of signal in presence of noise gives large error therefore increasing of order of approximation higher than two doesn't improve real accuracy.

For illustration lets assume $U(t) = \sin(wt)$, in this case difference between exact value $U(t)$ and one approximated using (7) or (9) can be calculated exactly. Result is shown in fig1. One can see that first order approximation gives 33% error at instability frequency while second order approximation accurate to 5%. We used (9) in our further analysis.

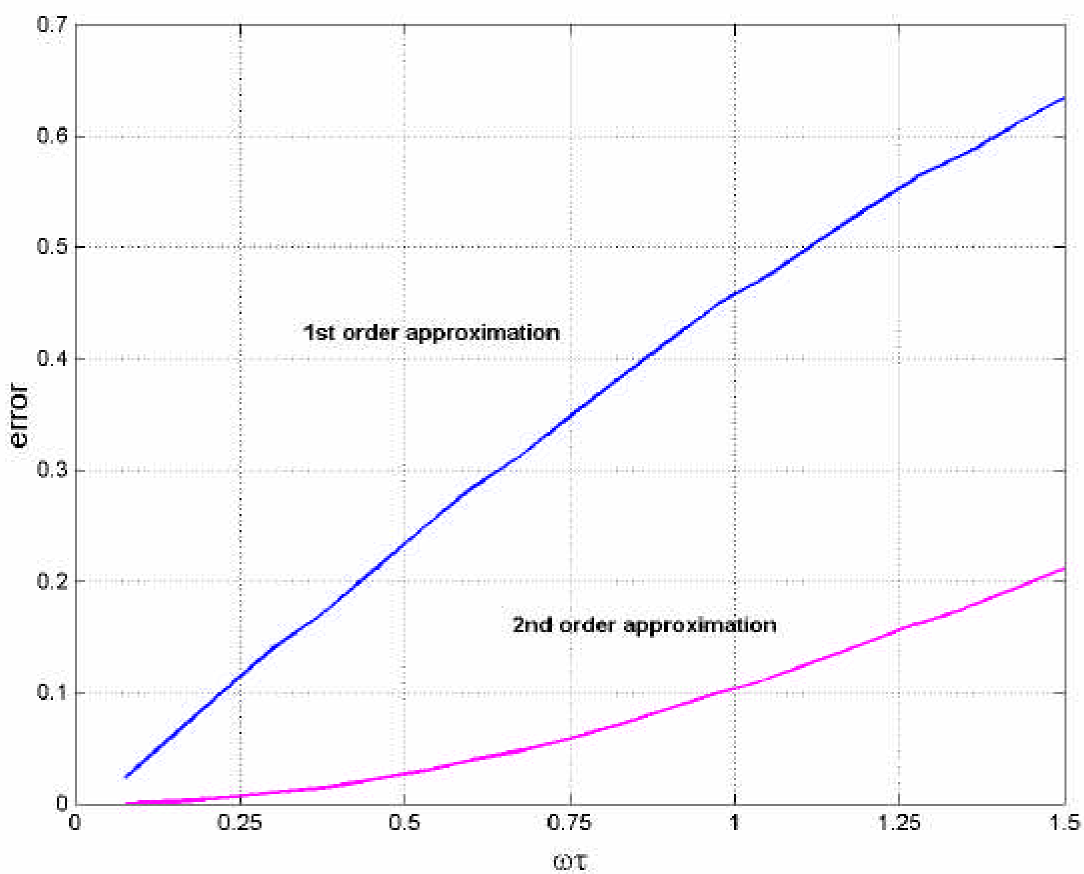

Figure 1. Difference between initial function f=sin(wt) and function restored after strip line transformation using 1st and 2nd order approximations.

Using (9) we calculate voltage on top and bottom BPM strip lines, u_t and u_b respectively. Then beam centroid displacement can be derived from its ratio [3]:

$$x[mm] = 1.4737 \cdot R - 0.00027 \cdot R^3, \tag{10}$$

where $R = 20 \cdot \log(\frac{u_t}{u_b})$. Numerical coefficients here are derived from calibration data for this particular BPM.

Due to beam movement along orbit data acquired by one BPM at fixed point do not represent centroid position of all points of the beam in one moment of time. Let $x_0(t,s)$ be beam centroid transverse position in beam frame of reference, where *s* is coordinate along orbit varying from 0 to orbit circumference $2\pi R$. Then BPM will measure in laboratory frame of reference transverse position $x=x_0(t + s/v, s)$, where v is beam velocity. We can write for Fourier transformation of x:

$$X(w,s) = F_t(x) = F_t(x_0(t+s/\mathrm{v},s)) = F_t(x_0)\cdot e^{iw\frac{s}{\mathrm{v}}} = X_0(w,s)\cdot e^{iw\frac{s}{\mathrm{v}}},$$

then we can derive $X_0(w)$

$$X_0(w,s) = X(w,s)\cdot e^{-iw\frac{s}{\mathrm{v}}}$$

and restore beam transverse position in beam frame performing inverse Fourier transformation

$$x_0(t,s) = F_t^{-1}(X_0(w,s)) = F_t^{-1}(X(w,s)\cdot e^{-iw\frac{s}{\mathrm{v}}})$$

The algorithm described above are implemented in MATLAB script. Result of BPM data processing for one turn is shown in fig.2. Blue line represents snapshot of beam centroid displasement on one turn, red line is the sum of top and bottom strip lines proportional to beam current.

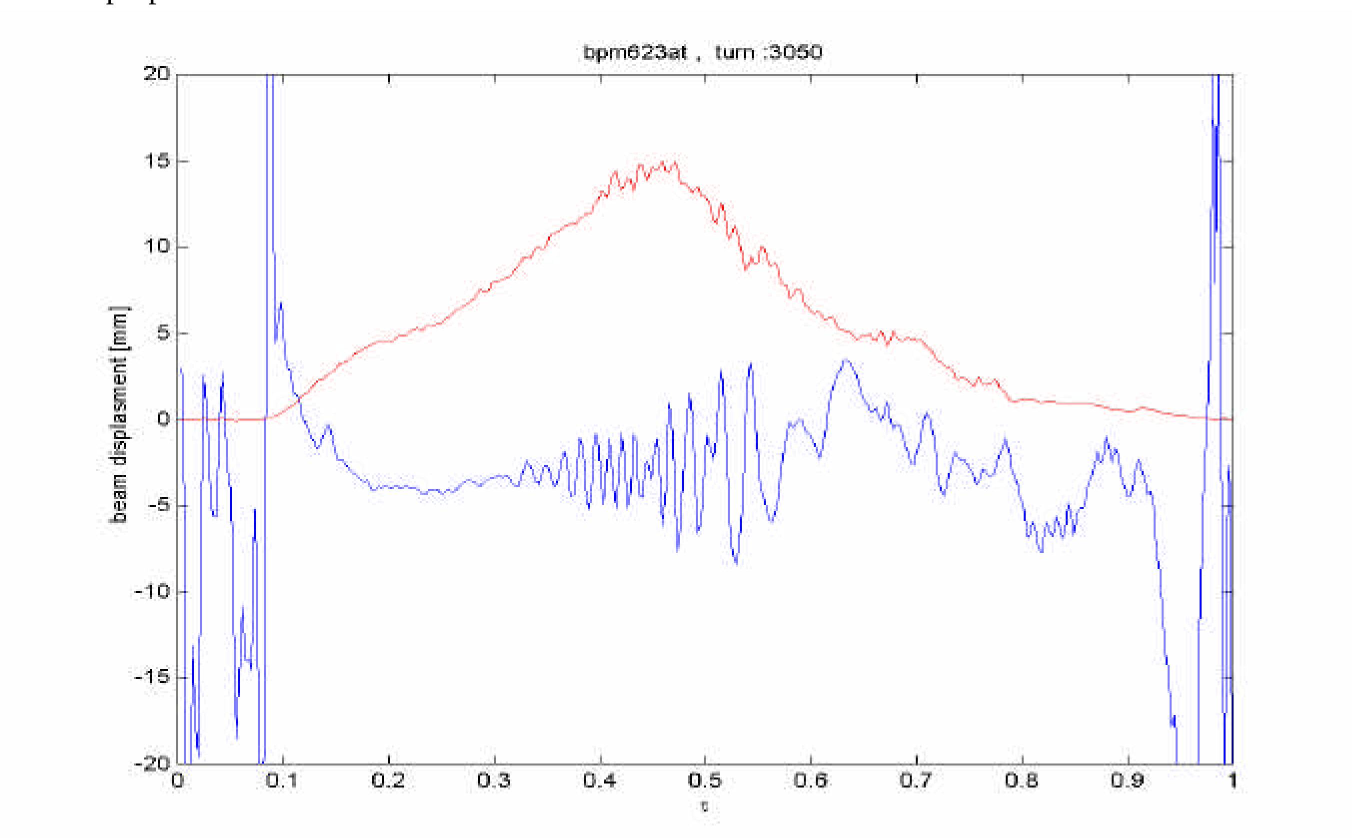

Figure 2. Beam displacements during one turn (blue line). Red curve is beam current profile.

In Figure 3 the amplitude of transversal oscillations of selected part of the beam versus number of turns is shown. The dashed straight line corresponds to the average growth of amplitude. The calculated increment of this instability is equal to 182 turns in this case.

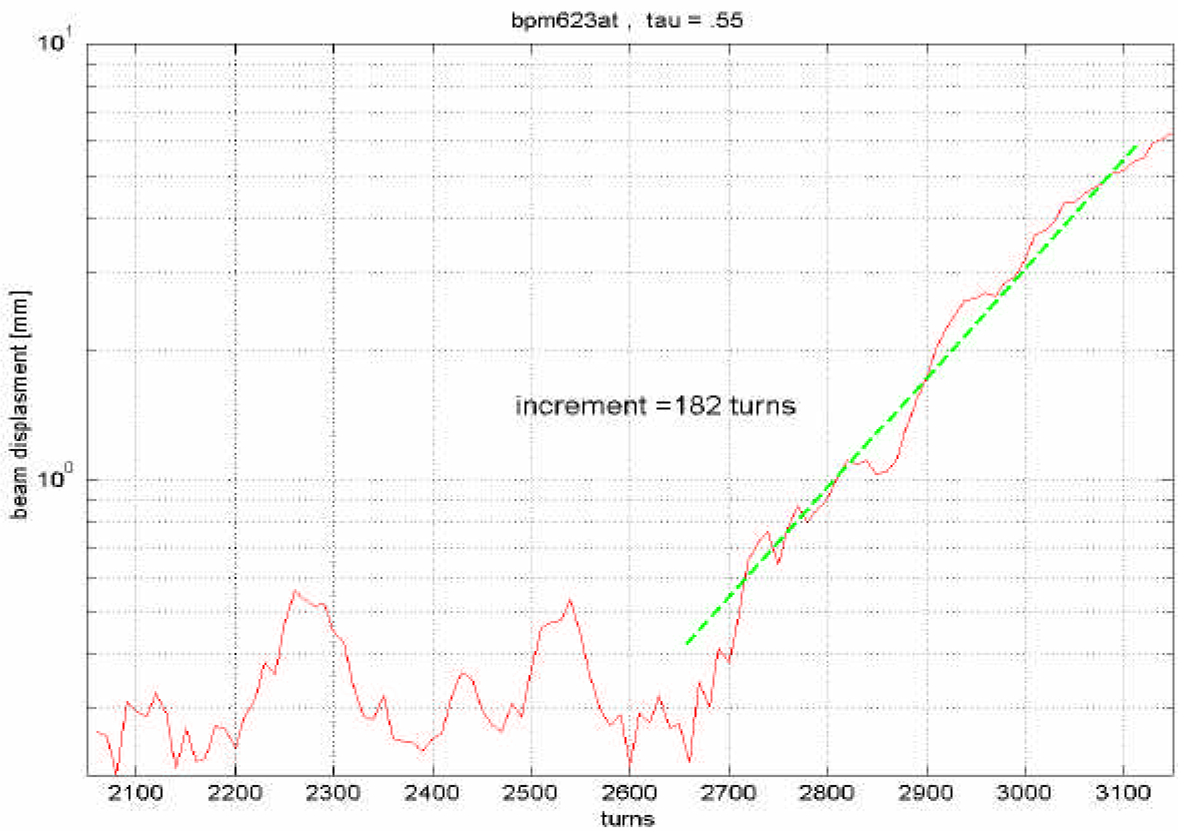

Figure 3. Amplitude of transverse oscillations versus number of turns.

III. Instability signatures as seen after 2D FFT transformation of the processed signals

After the extraction of the beam centroid offsets for each turn we form a two-dimensional matrices of numbers. This matrix has N columns and M rows where N is number of digitized points in each turn, M is number of turns to be processed (N=357, M=64 in further examples). Any single row represents centroid displacement data on one turn. BPM readings have large error on the edges of the beam, where current is small. Therefore we use a window (Hanning or square) for every row to select only data with the beam intensities not less than 10 % of its maximum. Rigorously speaking, we nullify the centroid coordinates in the gap to keep only informative part of the signal.

Every described above matrix corresponds to 64 beam turns. To see spatial harmonics and their frequencies we performed the 2D Fast Fourier Transformations with each matrix (in other words we develop beam centroid displacement in series of plane travelling waves in beam frame of reference).

The result of experimental signal processing with the instability present is shown in fig.4,5. Contour plot of sum signal proportional to beam current in coordinates (τ, turns) where $\tau = s/2\pi R$ is normalized distance along one turn is shown in fig.4.

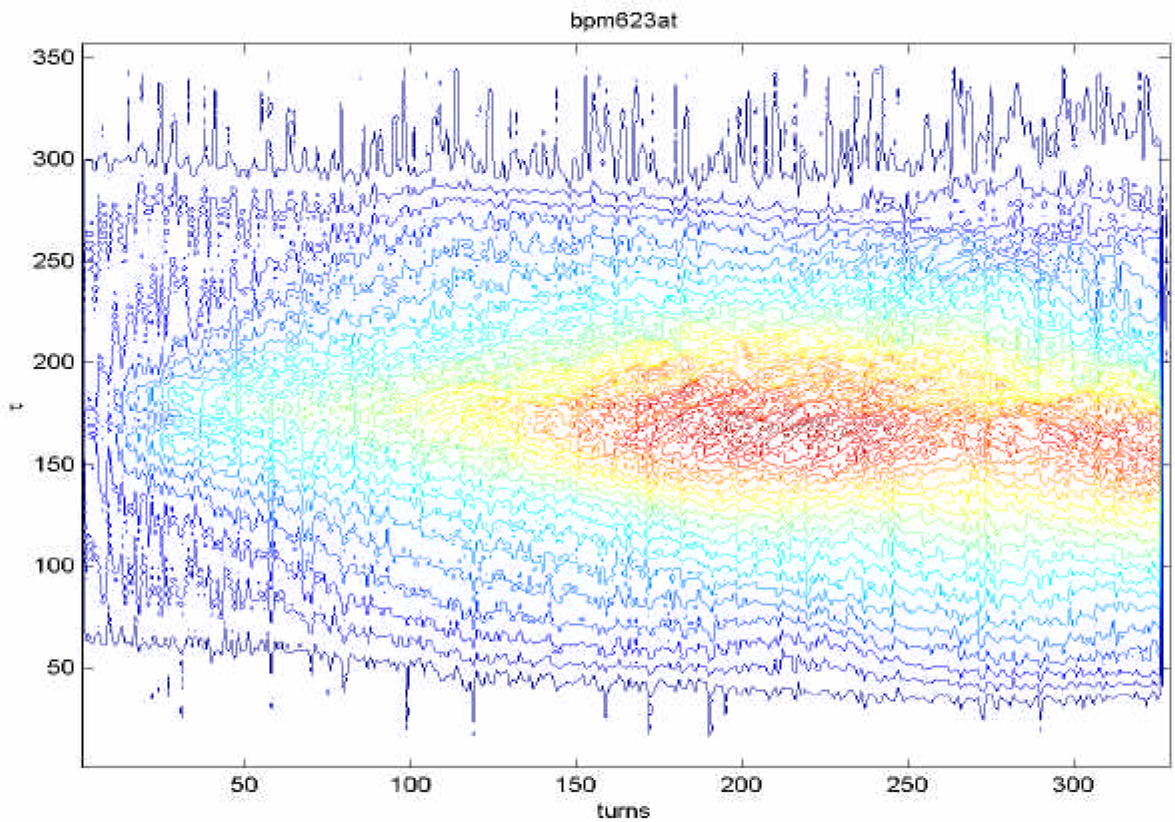

Figure 4. Contour plot of beam charge distribution in beam frame of reference in space-turns coordinates.

Fig 5. is a contour plot of harmonic amplitude. The peak heights increase from blue color to the green, yellow, and the red, which corresponds to the maximum of peak height. The horizontal axis is the frequency, expressed in terms of the revolution one. The vertical axis is the spatial harmonic number that is the circumference of the PSR, divided by the wavelength of harmonic oscillation. It's worth to note that only harmonics with opposite signs of ω and κ are present, which is well known feature of two-stream instability. The instability increments are larger than 64 turns, so we can see its developing by analyzing a sequence of the adjacent arrays of 64 turns. In fig.6 development of instability is shown (only upper left quarter of full spectrum is plotted).

In fig.6 one can see the development of the instability. Each subplot has the time difference of 100 turns with the previous one, with the a) subplot starting at 2400 turns from the beginning of the injection. The peaks with coordinates (0.18, k) correspond to the vertical betatron frequency. According to the "electron" explanation of this instability, the unstable harmonics should have harmonic number close to the ratio of the electron oscillation frequency with the revolution frequency and they should have frequency around the betatron one. That what we see at all pictures of this plot. As only oscillations with betatron frequency are observed we can fix this number and to see how space harmonics number changes with time when instability develops. Result is shown on fig.7 as spectrogram of harmonic amplitude in coordinates (turns, k_z). Spectrum for one turn is shown in fig.8.One can see some very interesting features of the instability:

1) Spectrum of space harmonics is discrete with $\Delta k_z \approx 5$. Note that this number is half of PSR periodicity.

2) Space wave number k_z decreases with number of turns linearly. It means that unstable harmonic can not be described by plane wave with constant frequency and wave number as in the case of coasting beam.

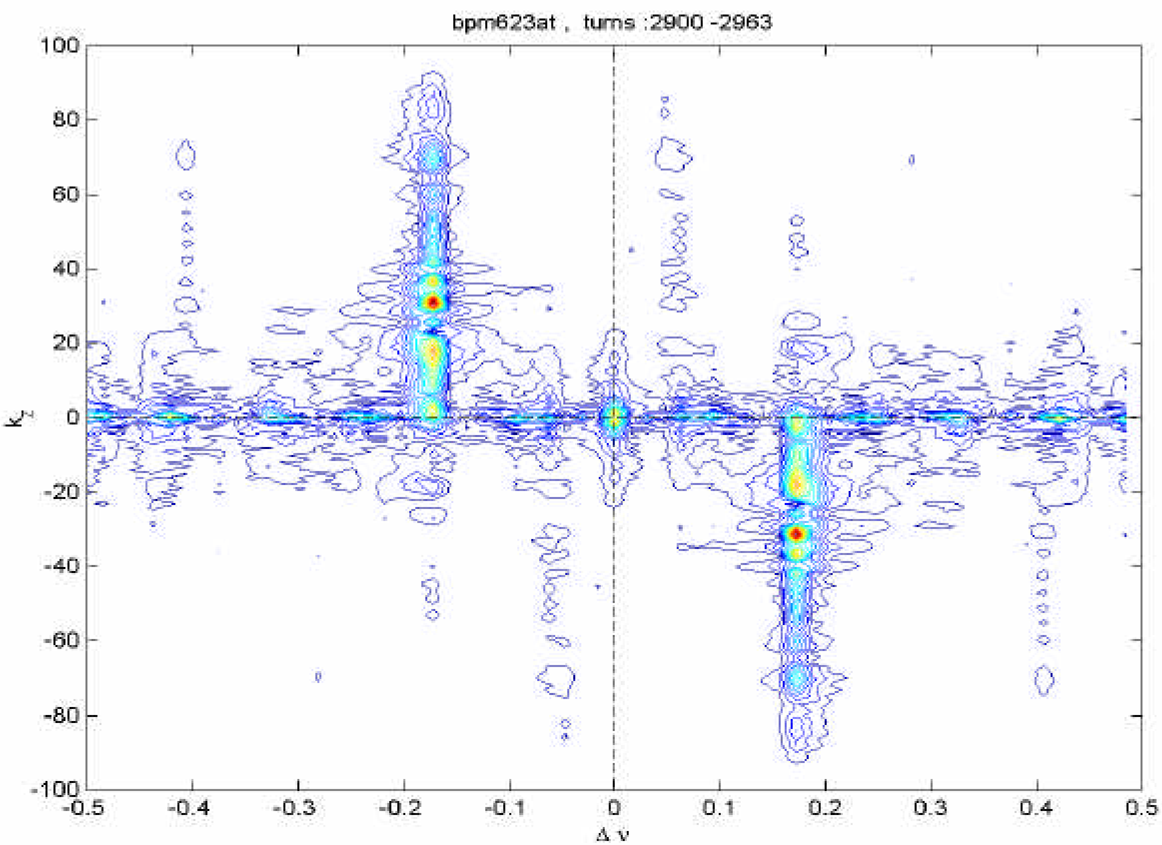

Figure 5. 2D Fourier spectrum of transverse beam oscillations.

IV. Conclusion.

We have presented a method of PSR fast BPM signal processing which allow to restore beam centroid transverse movement in time domain with high accuracy. It provides a powerful tool for PSR instability study. Some new results of beam movement analysis are presented.

References:

[1] Tai-Sen F. Wang, Technical Note: PSR-93-024, 1993
[2] King-Yuen Ng, Fields, Impedances and Structures
[3] J.D. Gilpatrick, Technical Note: LA-CP-97-21, 1997

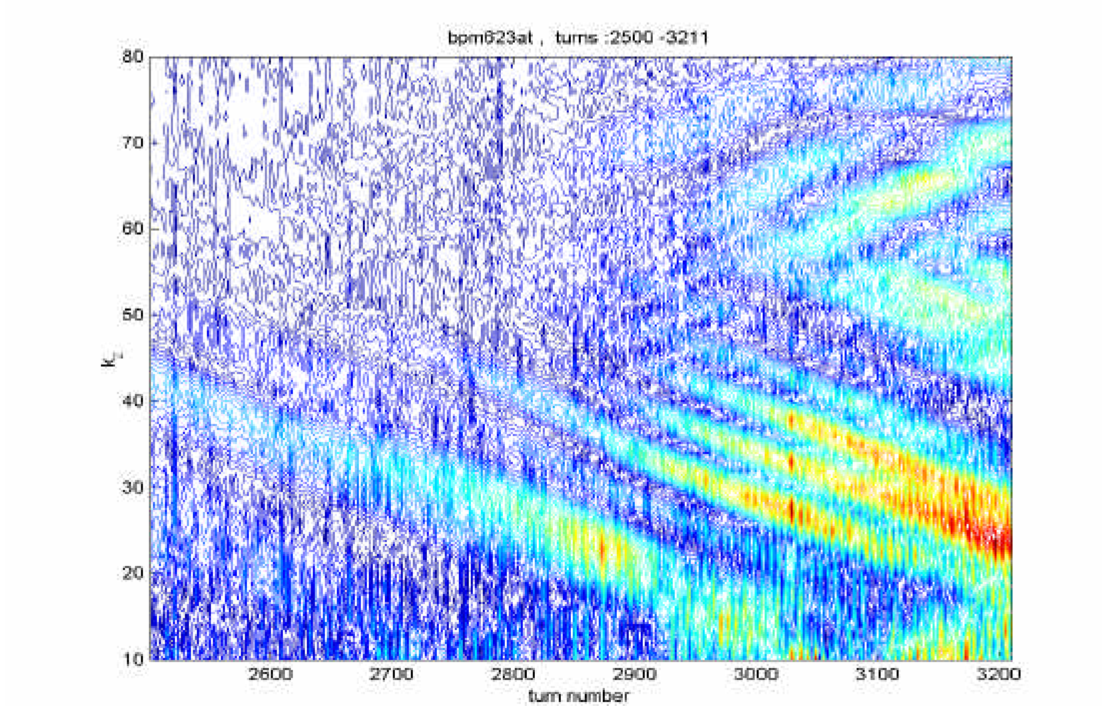

Figure 7. Space harmonics spectrogram of transverse beam oscillations.

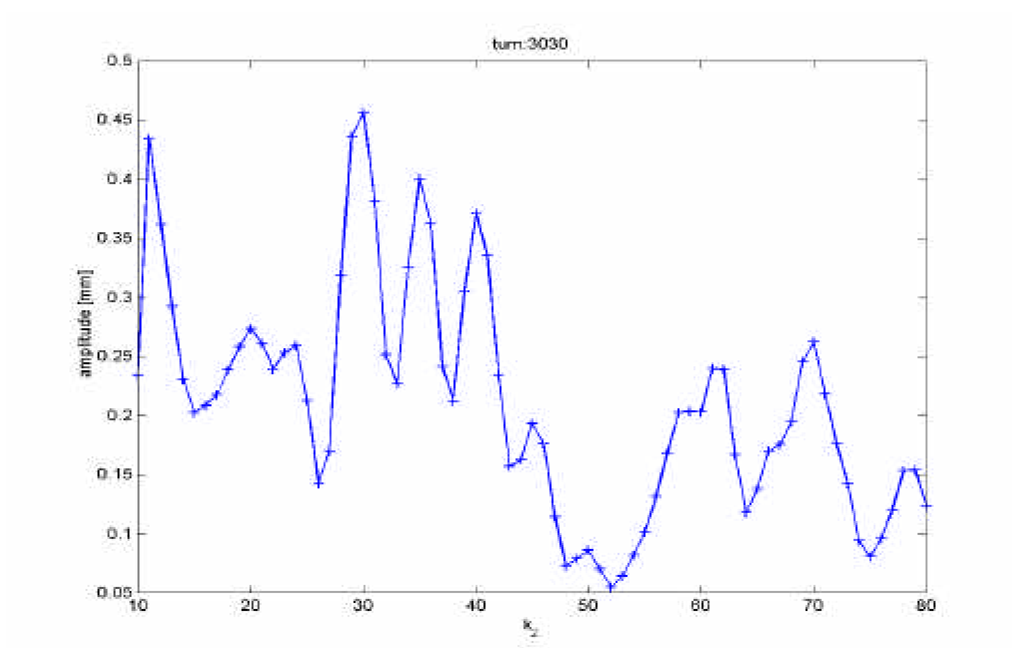

Figure 8. Space harmonics spectrum for one turn.

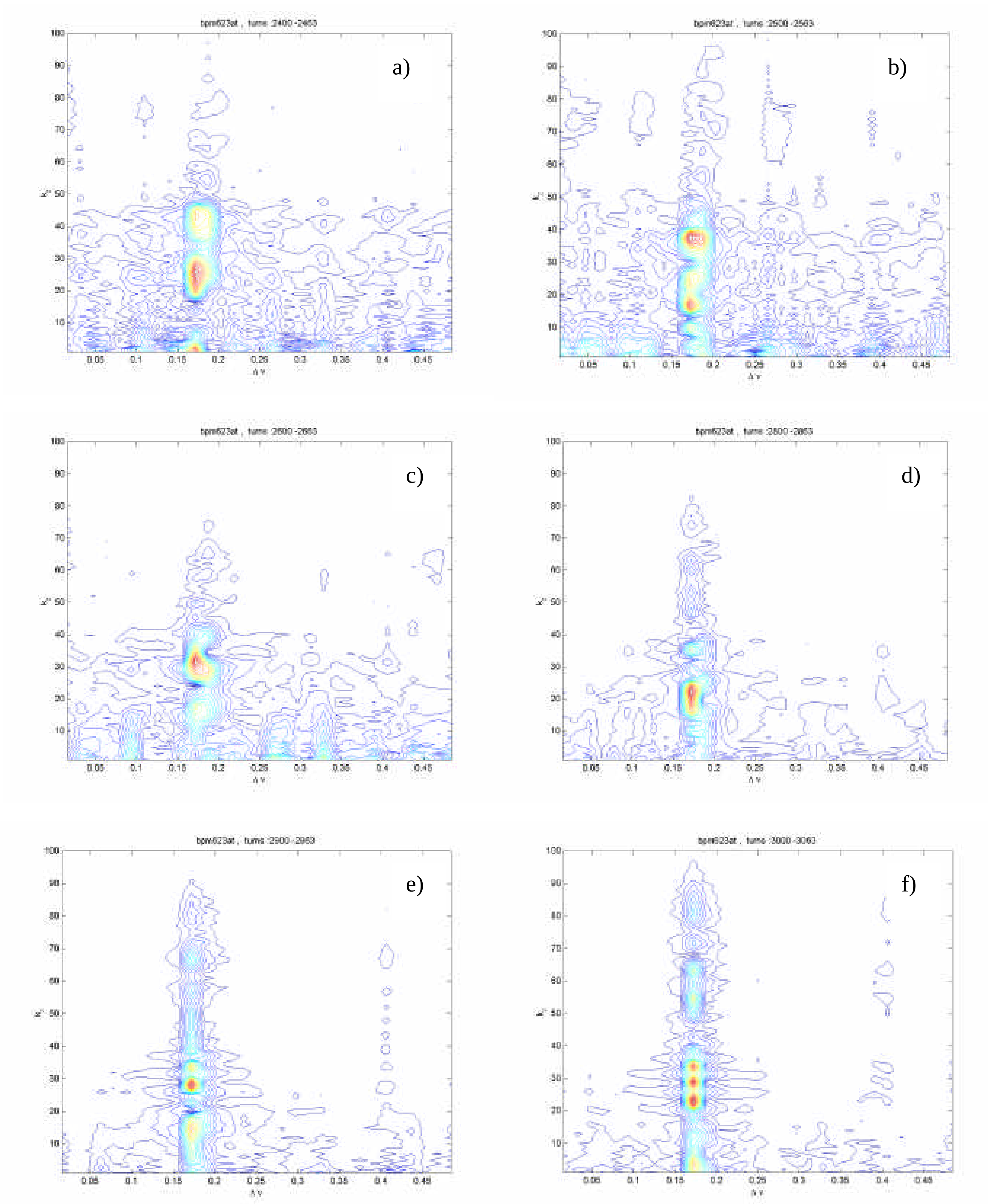

Figure 6. Contour plots of 2D Fourier transformation of beam transverse oscillations for successive arrays of 64 turns.