

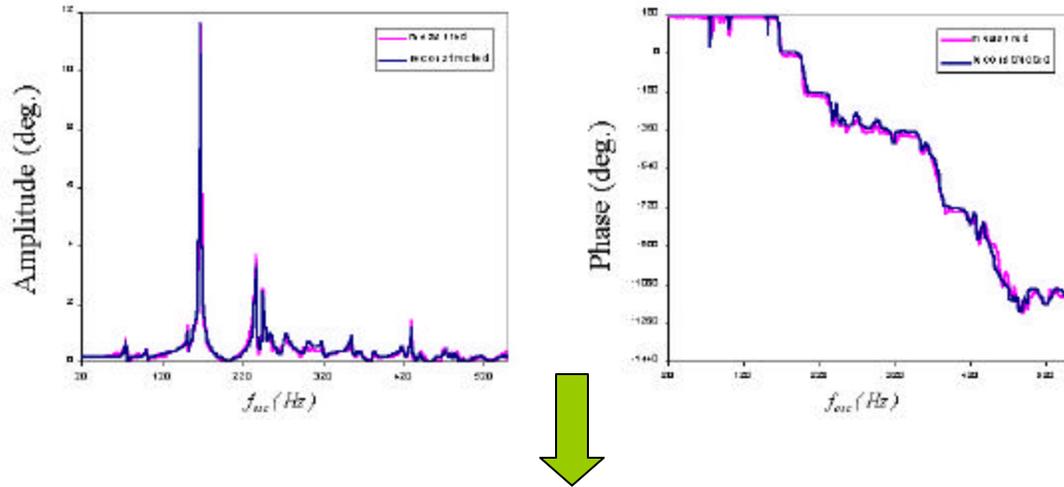
Study on Dynamic Detuning of SRF Cavity and its Application to the Compensation by Piezoelectric Tuner

Marc Doleans

02/25/2003

- SC Cavities in pulsed mode operation experience dynamic detuning due to time varying Lorentz forces on the cavity surface.
- The cavity voltage must be kept constant during the beam pulse.
- Need to minimize or compensate for the detuning:
 - Passive** - Reenforcement of the cavity structure
 - Active** - RF control = Additional RF power consumption
 - Compensation by piezoelectric

Measured and simulated Transfer function associated to the piezoelectric action.



Reconstruction of the Mechanical basis

Mode #	f(Hz)	Qm	km(Hz/mV)	Mode #	f(Hz)	Qm	km(Hz/mV)
1	75.5	200	-0.1	16	429.7	250	0.072
2	101.5	80	-0.05	17	440.5	100	0.04
3	152.8	260	-0.07	18	460	30	-0.05
4	168.5	120	-1.3	19	474	130	0.04
5	200	25	0.042	20	479	200	0.015
6	236.5	100	-0.45	21	485	100	0.04
7	245.1	100	-0.3	22	495	200	0.005
8	252.7	40	-0.24	23	517	50	0.06
9	272.6	40	-0.27	24	548	50	0.05
10	299.5	40	-0.15	25	558	250	0.05
11	319	100	0.07	26	563	300	0.015
12	355.5	100	0.1	27	570	300	0.025
13	365.7	40	0.12	28	587	160	0.1
14	383	100	-0.05	29	601.2	250	0.04
15	419	80	0.08				

Modeling/Understanding

- The excitation of the mechanical modes by the piezoelectric shows clear differences compare to the excitation by the Lorentz forces.

→ The coupling coefficients k_m can be of both signs for the piezo electric action.

→ Explains the behavior of the phase transfer function for the piezoelectric.

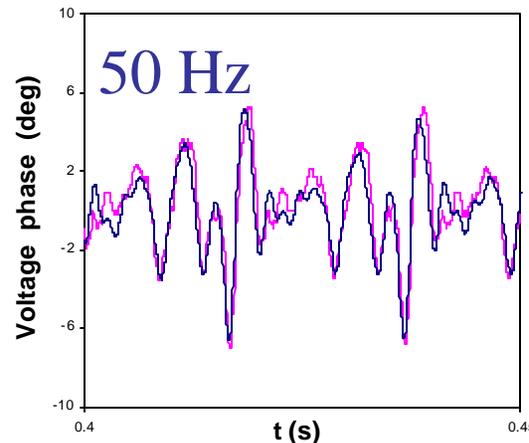
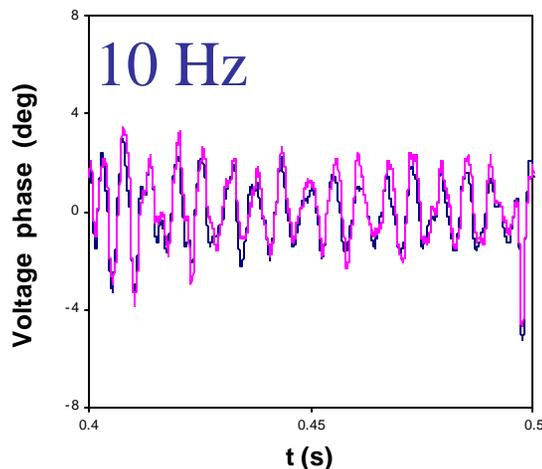
- The mechanical basis allows to have a virtual cavity for the simulations

→ Basis only reconstructed up to 600Hz.

How to relate the km of the mechanical basis to the single parameter K, (usually quoted to described the Lorentz detuning as $-K V^2$)?

- K is just the sum on all the coupling coefficients km
- K is relevant for static detuning cases only.

Simulated and measured voltage phase in the case of pulsed square waveform piezoelectric excitation



• The predictions using the mechanical basis matched well with the measurements

→ The model seems capable to simulate accurately the real piezoelectric action

→ Simulations can be used to study and determine adequate detuning compensation scheme.

• The mechanical basis allows to have a virtual cavity for the simulations

→ Basis only reconstructed up to 600Hz.

→ Tool ready for practical application !

Why are the coupling of the Lorentz forces and the piezoelectric different?

→ Possible to see the cavity surface as a vibrating string. The coupling of a given force to a given mechanical mode is determined by the projection of this force on the mode shape.

→ Since the Lorentz forces are distributed along the entire cavity surface whereas the piezoelectric action is only local, their coupling to the cavity structure can be very different.

Why the coupling coefficients are always of the same sign for the Lorentz forces action?

→ Very particular case. Easily understandable with the vibrating string qualitative model.

→ Due to the fact that the coupling coefficients are the product of the projection of the force on the mode shape & the projection of the detuning sensitivity on the mode shape.

$$Force \propto H^2 - E^2$$

$$Freq.sens \propto E^2 - H^2$$

→ Product always have the same sign in the case of Lorentz forces.

→ Not true for the Piezoelectric action (Force independent on the field)

If the couplings of Lorentz Forces and Piezoelectric action are so different is it still possible to compensate the Lorentz detuning using piezoelectric?

→ Ideally **yes**. (See details after)

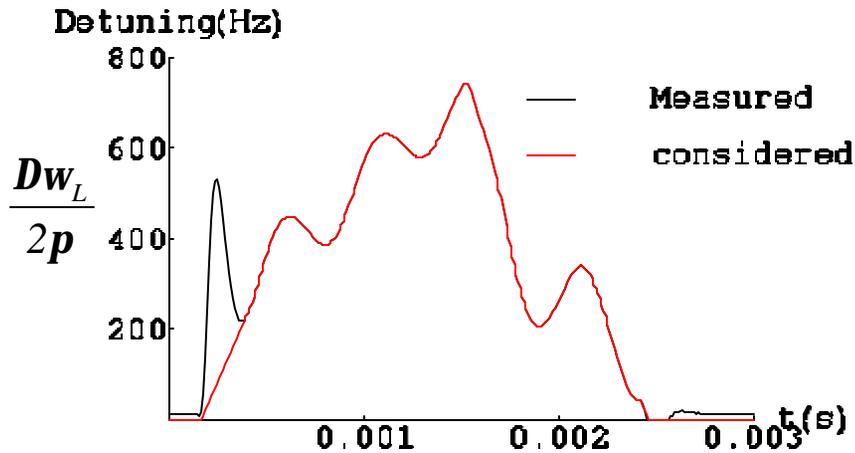
→ Practically, **yes**. But practical limitations (limited range of motion for the piezoelectric, necessity to have a piezoelectric input voltage waveform as simple as possible, knowledge of the Lorentz detuning only during RF pulse...) make a perfect compensation unlikely.

→ Not a problem, the goal is to avoid large dynamic detuning, approximate compensation is enough.

→ TESLA and JLAB prove experimentally that approximate compensation is possible.

→ **Optimization/Automization of the compensation scheme is the next step.**

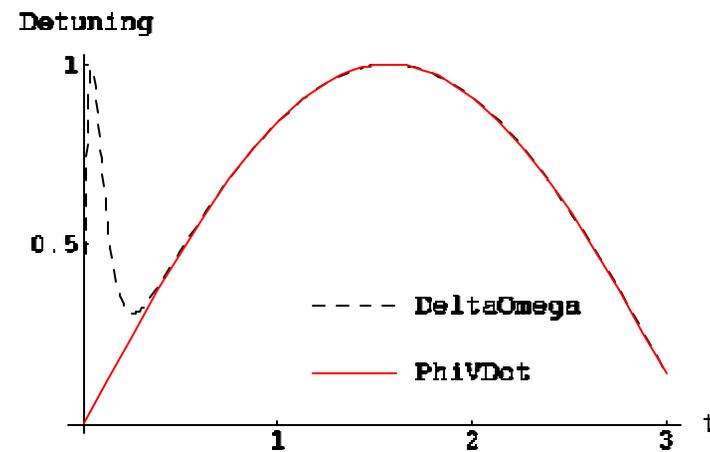
Measurement (JLAB)



- Cavity operated 60 Hz at nominal gradient.
- The early part of the detuning is not considered in the following because it is originating from a parasitic term.
- The Lorentz detuning is unknown after 2.5 ms because there was no field in the cavity between consecutive RF cycles. (No reason to be equal to zero)
- Signal repeatable 60 HZ as the rep. rate

→ Signal to compensate with piezoelectric

Simulation



At the transient, when the voltage is small a parasitic term can be large even for small value of the phase loop.

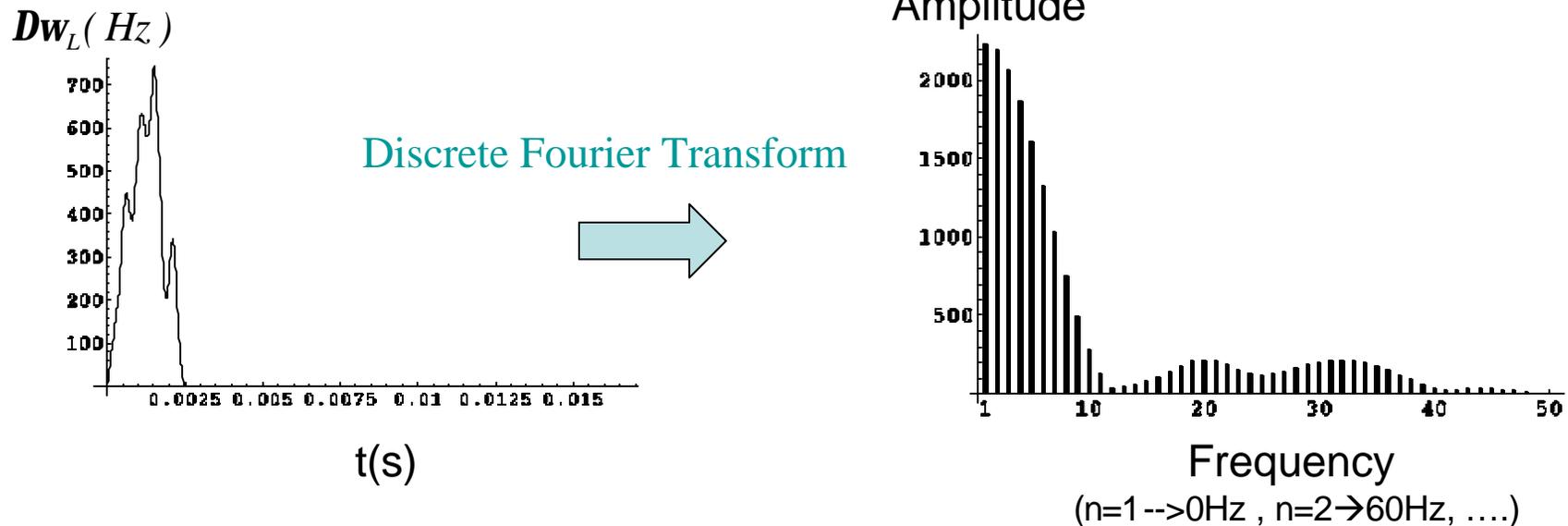
$$\left(\dot{F}_V(t) = Dw(t) + w_{1/2} \frac{R_L I(t)}{V(t)} \sin q_l \right)$$

The goal is to find an input voltage waveform for the piezo to Compensate for the detuning during the beam pulse, and if possible during the turn on transient.

The detuning generated by the piezo should ideally be the opposite of the Lorentz detuning for these periods of time.

The Lorentz detuning is repeatable 60 Hz.

→ Only composed of 60 Hz harmonics Lines



→ The detuning generated by the piezo should also be repetitive 60Hz.

→ According to the model it follows that the input voltage signal should also be repetitive 60 Hz.

→ The Solution for the Piezo Input Voltage should be searched as a superposition of 60 Hz Harmonics.

$$\Delta\omega_L(t) = \sum_{n=0}^{\infty} \Delta\omega_{L,n} \cos(n\omega_{rep}t + \phi_{L,n}) \quad (2.47)$$

$$V_P(t) = \sum_{n=0}^{\infty} V_{P,n} \cos(n\omega_{rep}t + \theta_{P,n}) \quad (2.48)$$

According to Section A.6, such forcing function will produce, in the steady state, a piezoelectric detuning function of the type

$$\Delta\omega_P(t) = \sum_{n=0}^{\infty} \Delta\omega_{P,n} \cos(n\omega_{rep}t + \phi_{P,n}) \quad (2.49)$$

To compensate the Lorentz detuning it follows that $\Delta\omega_{P,n} = \Delta\omega_{L,n}$ and $\phi_{P,n} = \phi_{L,n} + \pi$. From Section A.6, the solution for each harmonic of the forcing function V_P is obtainable by summation on all the mode contributions. Using the result of Eq. (A.40) gives

$$\Delta\omega_P(t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} V_{P,n} \frac{\Omega_{P,m}}{\omega_n} \frac{k_{P,m} Q_{P,m}}{\sqrt{1 + \tan^2 \psi_{P,n,m}}} \cos(n\omega_{rep}t + \theta_{P,n} + \psi_{P,n,m} - \frac{\pi}{2}) \quad (2.50)$$

where the parameters of the m^{th} mechanical mode related to the piezoelectric action are noted $\Omega_{P,m}$, $k_{P,m}$, $Q_{P,m}$ and where the phase $\Psi_{P,m}$ follows the same definition than in Eq. (A.40). Combining the results of Eq. (2.50) and Eq. 2.49 leads to

$$\begin{aligned} a_n &= \sum_{m=1}^{\infty} \frac{\Omega_{P,m}}{\omega_n} \frac{k_{P,m} Q_{P,m}}{\sqrt{1 + \tan^2 \psi_{P,n,m}}} \cos(\psi_{P,n,m} - \frac{\pi}{2}) \\ b_n &= \sum_{m=1}^{\infty} \frac{\Omega_{P,m}}{\omega_n} \frac{k_{P,m} Q_{P,m}}{\sqrt{1 + \tan^2 \psi_{P,n,m}}} \sin(\psi_{P,n,m} - \frac{\pi}{2}) \end{aligned} \quad (2.51)$$

$$\Delta\omega_{P,n} = V_{P,n} \sqrt{a_n^2 + b_n^2}$$

$$\phi_{P,n} = \theta_{P,n} + \arctan\left(\frac{b_n}{a_n}\right)$$

It eventually gives for the amplitude and phases of the n^{th} harmonic of the piezoelectric input voltage

$$\begin{aligned} V_{P,n} &= \frac{\Delta\omega_{L,n}}{\sqrt{a_n^2 + b_n^2}} \\ \theta_{P,n} &= \phi_{L,n} + \pi - \arctan\left(\frac{b_n}{a_n}\right) \end{aligned} \quad (2.52)$$

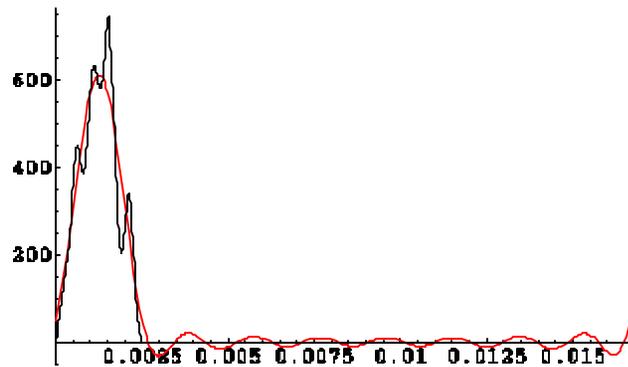
Input voltage for the piezoelectric can be found for the model using the mechanical basis parameters.

→ The Lorentz detuning is compensated harmonics by harmonics using the superposition principle.

→ Doesn't require that the mechanical parameters for the Piezoelectric and for the Lorentz forces be equal

Since the mechanical basis is only known (for now) up to 600 Hz, only the first 10 harmonics of 60 Hz will be considered in the following examples.

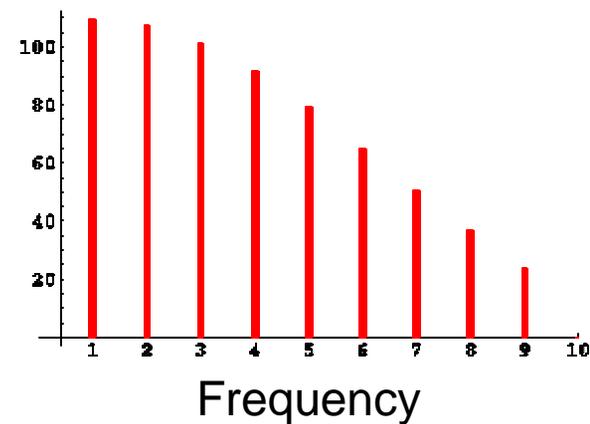
→ Extendable to higher frequency



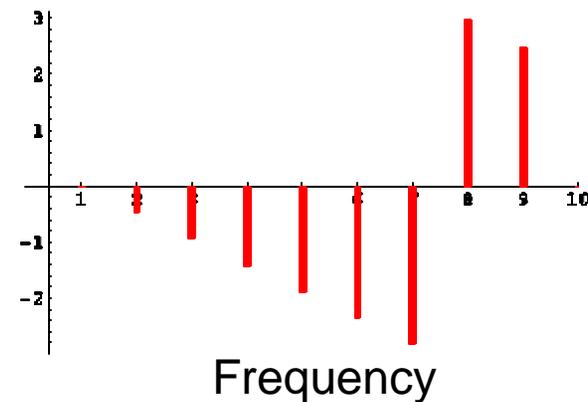
Discrete FFT



Amplitude

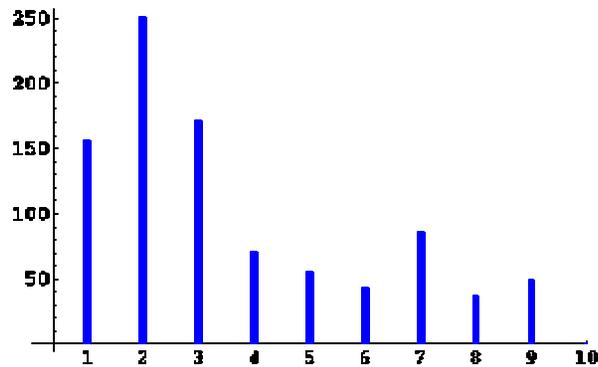


Phase



→ The red function is the detuning that will be tried to be compensated in the first example.

Amplitude

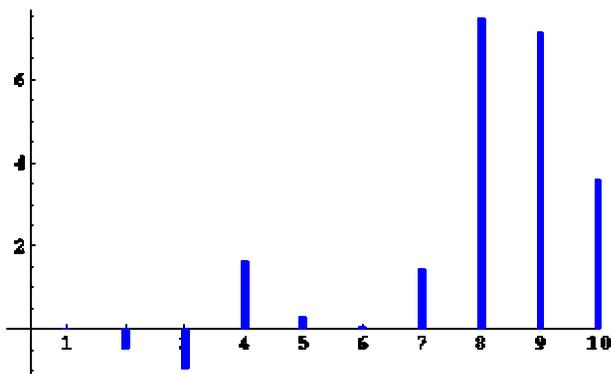


Frequency

The amplitudes and the phases for the first 10 harmonics are found according to the previous formula.

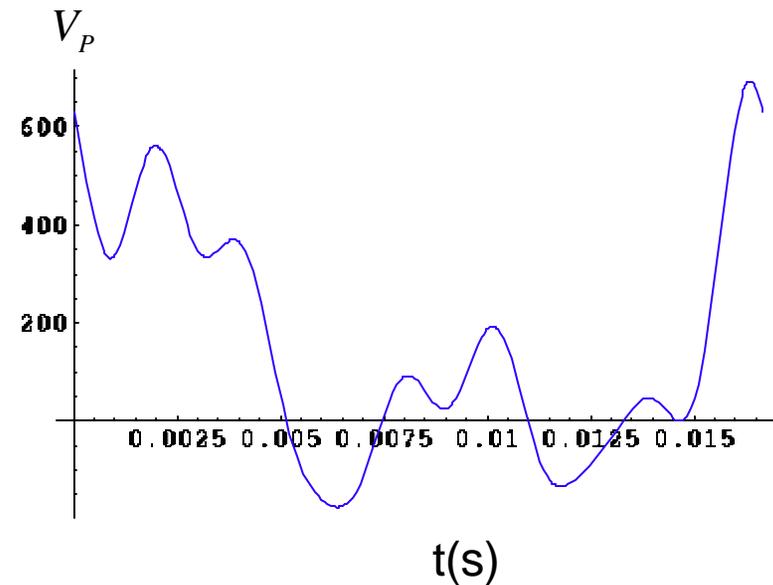
→ The forcing input voltage for the piezoelectric is reconstructed from these fourier components and is of course periodic 60 Hz.

Phase



Frequency

Inverse FFT

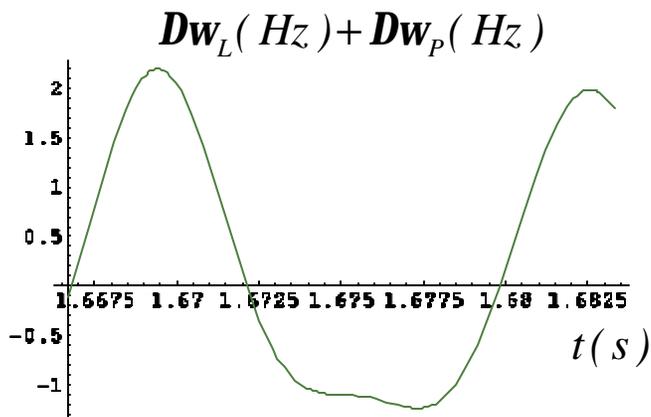
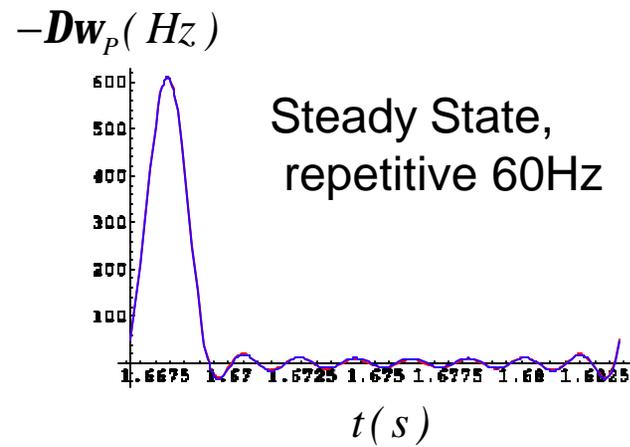
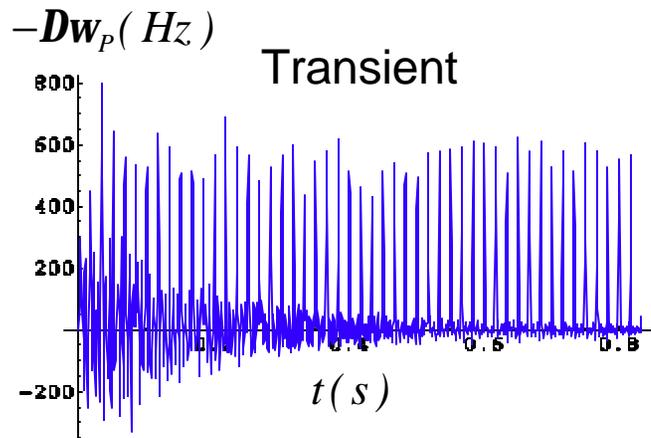


→ The input voltage signal can be used in the model to check if the detuning from the piezo is correct.

The forcing function is used as an input in the model.

→ the detuning generated by the piezo shows a transient behavior and stabilizes in steady state

→ The detuning in steady state should be equal and opposite of the targeted initial detuning function. (!)

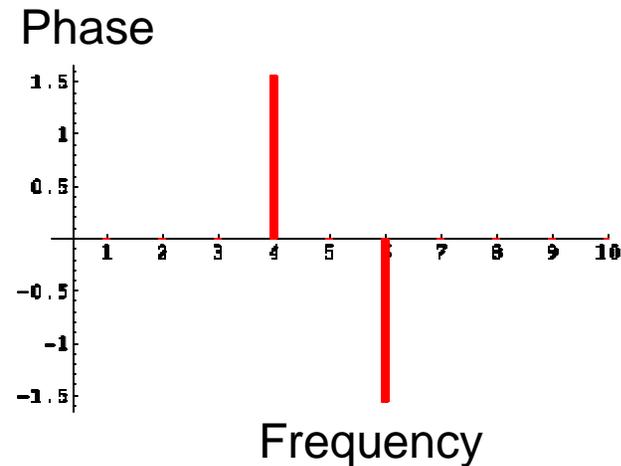
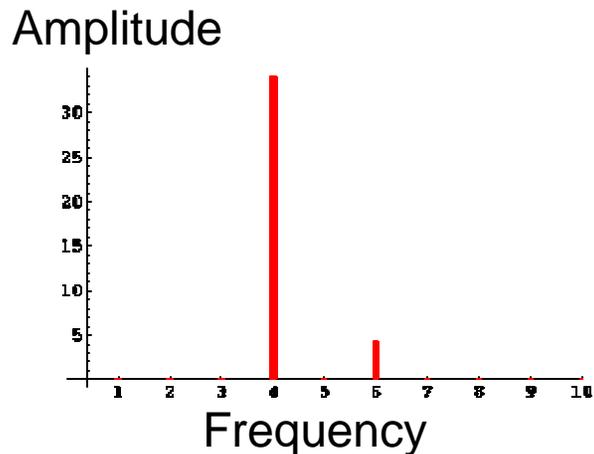
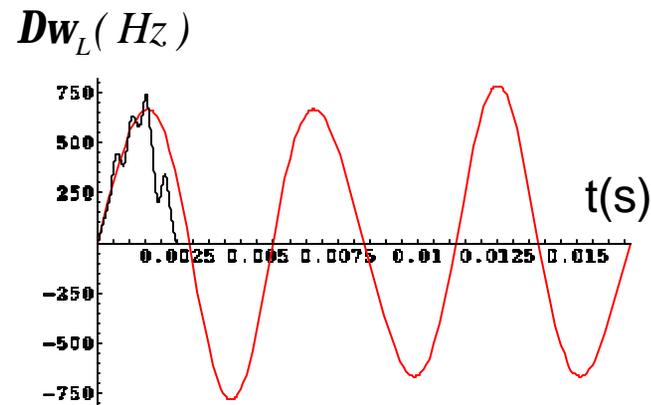
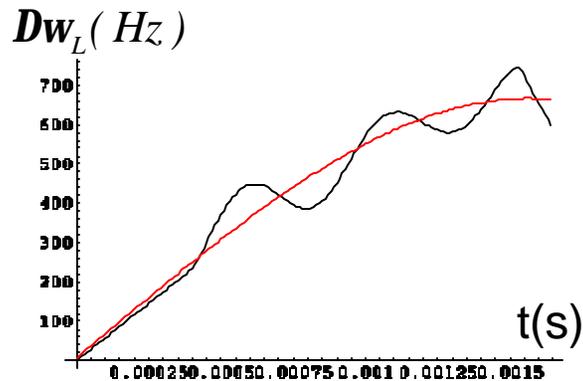


The detuning generated and the initial detuning are summed and the result function is found close to zero as expected.

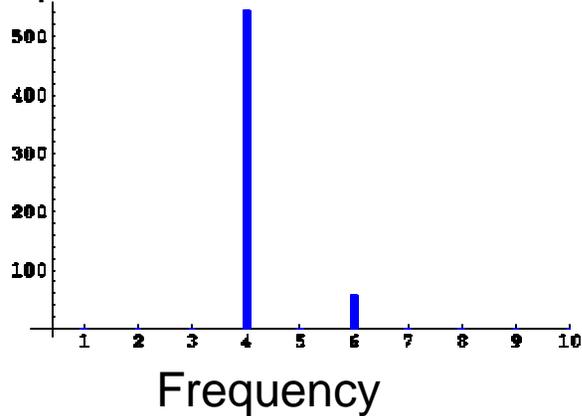
→ Compensation scheme is working and can be simplified (see next)

The previous scheme was using 10 harmonics. It is possible to concentrate only on the Lorentz detuning part during the beam pulse and the turn on transient.

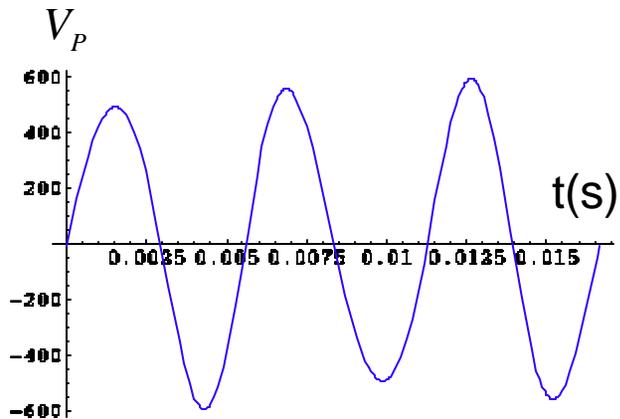
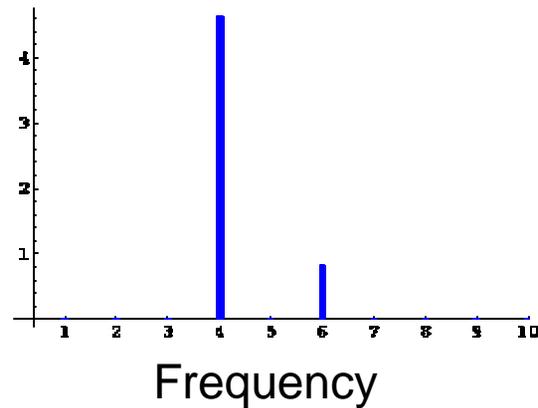
→ Looking at the detuning shape it appears that using a simple harmonics (180Hz) could be enough to have a good approximate compensation.



Amplitude



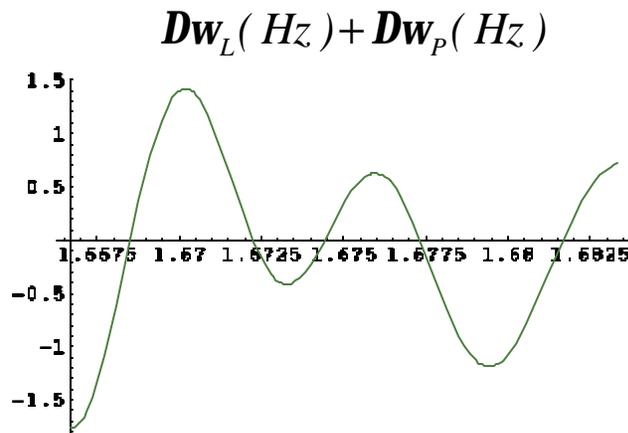
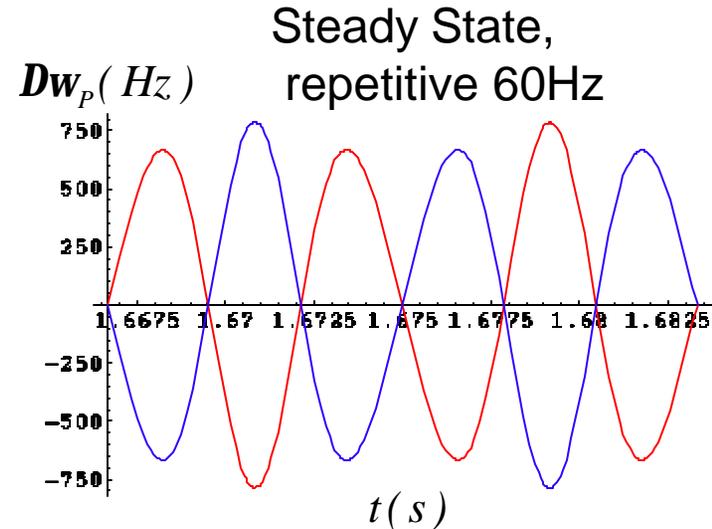
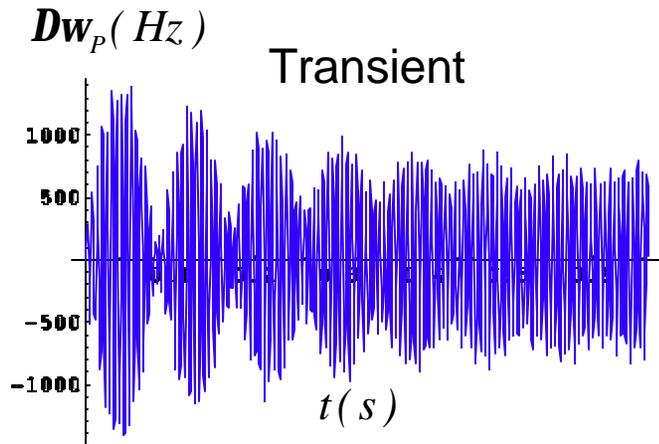
Phase



→ The forcing input voltage is a very simple function (CW)

→ Usable in practice

→ The input voltage signal can be used in the model to check if the detuning from the piezo is correct.



(Red signal is the function trying to be compensated)

The detuning generated and the initial detuning are summed and the result function is found close to zero as expected.

→ Simple Compensation scheme is working

- Model is interesting to explain real measurements (static K Vs. modal km, coupling coefficients for the Lorentz detuning of same signs, parasitic term at the turn on transient.....)
- Model can be used to study optimization process and more (RF control, frequency tracking using field measurement...)
- Simple compensation scheme using a single harmonic of the repetition rate for the piezoelectric input voltage seems a reasonable scheme to approximately compensate for the Lorentz detuning
- other waveforms (Trapezoidal...) can also be studied but are not as straightforward.
- The parameters for the piezoelectric input voltage were find using the reconstructed mechanical basis.

→But, in practice it can be done much more simply by using the information of the 60 Hz line(s) of the Piezoelectric Transfer function.



$$V_{P,n} = \frac{\Delta\omega_{L,n}}{\Delta\omega_a(n\omega_{rep})} V_a$$
$$\theta_{P,n} = \phi_{L,n} + \pi - \phi_a(n\omega_{rep})$$