A PROTOTYPE RF COMB FILTER FEEDBACK FOR THE FERMILAB MAIN INJECTOR

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Abstract

The Fermilab Main Injector has steadily been entering into an era of heavy beam loading. The advent of slip-stacking for pBar production required the addition of direct RF feedback and feed forward for transient beam loading compensation to reach intensities of 3.3E13 protons per cycle. Intensities from 4.5E13 to 8E13 for the future neutrino program are foreseen, which require not only transient beam loading compensation but also longitudinal coupled bunch growth rate reduction. Although an ideal feed-forward system can address both issues, a comb filter feedback system offers the advantages of feedback to complement the present feed-forward system. Building upon the foundation laid by the pioneers and practitioners of comb filter feedback, a prototype single-peak comb filter feedback loop has been designed and tested on a Main Injector RF station using present day digital technology. The design architecture with practical solutions to common issues is presented along with experimental results.

INTRODUCTION

The development of comb filter feedback in circular machine RF systems is well documented [1-4]. Due to the low synchrotron tune (max Q_s~0.01) in the Fermilab Main Injector, a single-peak comb filter offers reduction of both (a) transient beam loading at the revolution harmonics and (b) the longitudinal coupled bunch mode driving impedance of the fundamental RF system. The principle is to add additional feedback around the RF cavity, as shown in Fig. 1, near harmonics of the beam revolution frequency, f_{rev}, via a comb filter with a delay. The feedback transfer function is

$$H(j\omega) = \frac{G}{1 - K e^{-\frac{j\omega}{f_{rev}}}} \cdot e^{-j\omega T_d} \qquad (1)$$

with gain parameter G, comb shape parameter K, delay T_d, $j = \sqrt{-1}$, and ω is radian frequency.

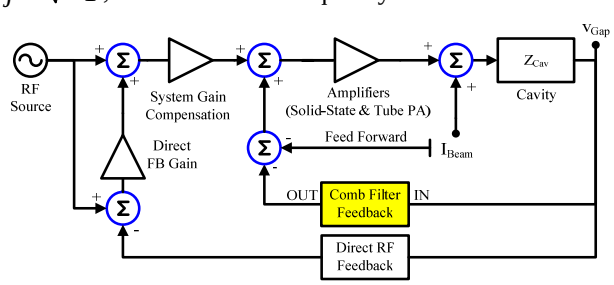

Figure 1: Simplified MI RF station block diagram.

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COMB FILTER ARCHITECTURE

A simplified block diagram of the comb filter design is shown in Fig. 2; signal flow is from right to left similar to its placement in Fig.1. The cavity RF gap monitor is fed to a bandpass filter (BPF) before being downconverted to an intermediate frequency (IF) with a local oscillator (LO) to prevent out-of-band noise or any signals due to cavity higher-order modes from being mixed down or aliased into the nyquist band. An anti-aliasing low-pass filter (LPF) is included on the ADC input.

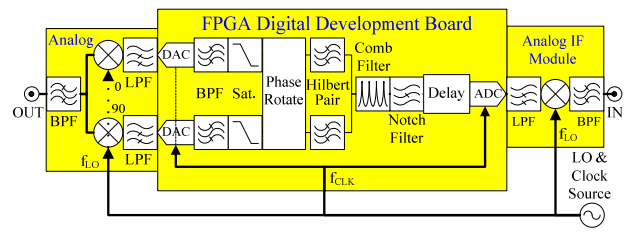

Figure 2: Comb filter design block diagram.

The signal processing within the field programmable gate array (FPGA) includes: (1) a delay element used for realizing a near one-turn delay, (2) a narrow notch filter at the IF frequency to eliminate the fundamental RF signal, (3) the comb filter, (4) a digital quadrature splitter realized with a set of BPFs whose outputs approximate a Hilbert pair over the passband for realizing phase rotations and single-sideband (SSB) upconversion, (5) a phase rotation for optimizing the loop stability in conjunction with the near one-turn delay, (6) a set of saturation blocks for protection against overflow, and (7) a set of BPFs to eliminate the spectral content caused by any saturation and to reduce the average noise power.

The quadrature output signals from the DACs are fed to a SSB upconverter via a pair of reconstruction LPFs. The final output BPF reduces any LO leakage and the image frequencies due to imperfect SSB conversion.

Notch Filter

Eliminating the fundamental RF signal with the notch filter greatly simplifies the integration of the new comb filter loop. It minimizes the impact on the existing fundamental loops and eliminates the new loop from becoming the dominant fundamental RF controller; hence there is no need for generating a reference signal for the new loop (with all the associated practical complications in swept RF systems of maintaining its proper phase relationship to the feedback as in a direct RF FB loop).

The digital notch filter [5] is formed using an all-pass filter as arranged in Fig. 3. The all-pass filter is constructed using a structurally lossless, second order, infinite impulse response (IIR) filter shown in Fig. 4 with

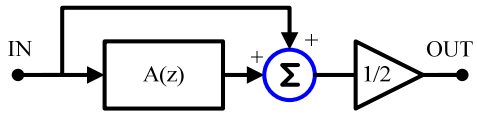

Figure 3: Notch filter constructed with all-pass filter *A(z)*.

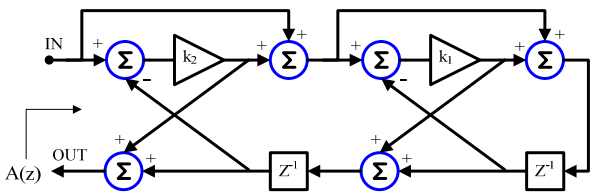

Figure 4: Second-order all-pass filter *A(z)*.

$$A(z) = \frac{k_2 + k_1(1+k_2)z^{-1} + z^{-2}}{1 + k_1(1+k_2)z^{-1} + k_2 z^{-2}} \qquad (2)$$

where $k_1 = -\cos\left(2\pi \frac{f_{notch}}{f_{CLK}}\right)$ controls the notch frequency, f_{notch}, and $k_2 = \frac{1-\tan\left(\pi \frac{BW_{notch}}{f_{CLK}}\right)}{1+\tan\left(\pi \frac{BW_{notch}}{f_{CLK}}\right)}$ controls the notch bandwidth, BW_{notch}. When using binary fixed-point arithmetic in the FPGA, a natural choice is k_1 = -1/2 to give $f_{notch} / f_{CLK} = 1/6$. A practical choice for k_2 is *n/p* where *n* is an integer and *p* is a power of two. As k_2 approaches unity, BW_{notch} approaches zero and the poles of *A(z)* approach the unit circle. Measurement results at the IF bandwidth with k_1 = -1/2 , k_2 = 255/256, and f_{CLK} = 43.5MHz are shown in Fig. 5. Greater than 45dB rejection at f_{notch}, $BW_{notch} \cong 27kHz$, and <9deg deviation from linear phase at +/-f_{rev} ~(+/-90kHz) is achieved.

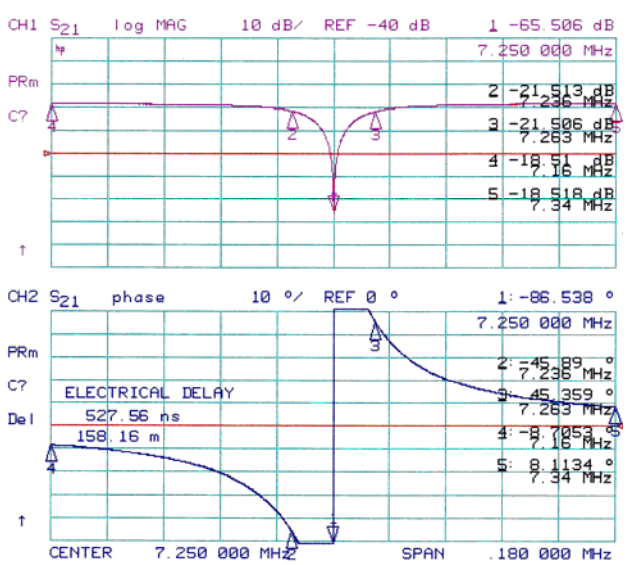

Figure 5: Notch filter measurements k_1=-1/2, k_2=255/256, f_{CLK}=43.5MHz.

Comb Filter, IF, LO, and Clock Frequency

Since downconversion shifts an original input spectrum *S(f)* to $S(f - f_{LO}) + S(f + f_{LO})$, with f_{LO} the LO frequency, the choice of downconverting to an IF as opposed to downconversion to baseband uses twice the bandwidth but requires only one ADC since the upper and lower sidebands are not forced to lay on top of each other, thus eliminating the need for quadrature downconversion to maintain information from both sidebands. It also opens the door for phase (and/or magnitude) equalization that can be asymmetric about the fundamental as opposed to the symmetric equalization forced by the sidebands overlapping in a baseband architecture as observed in [4].

The MI RF frequency, $f_{RF}=hf_{rev}$, sweeps ~300kHz from ~52.8114 MHz to ~53.104 MHz; harmonic number *h* is 588. The z-transform of the IIR digital filter equivalent to Eq. 1 is easily found by substituting $z = e^{\frac{j\omega}{f_{CLK}}}$ to give

$$H(z) = \frac{G}{1 - K\, z^{-\frac{f_{CLK}}{f_{rev}}}} \cdot z^{-T_d \cdot f_{CLK}} \quad . \qquad (2)$$

An integer number of delays for the denominator *z* term is achieved if the sample clock frequency, f_{CLK}, also sweeps and is made equal to a harmonic of f_{rev}, thus allowing the comb filter peaks to track the f_{rev} harmonics A beautiful solution is realized by using a phase-lock-loop to generate both f_{CLK} and $f_{LO} = f_{RF}(L/M)=hf_{rev}(L/M)$ with *M*=7, a factor of *h*=588, and *L*=6. This results in (a) the comb filter tracking the frequency sweep and (b) f_{RF} falling onto $f_{notch}=(1/6)f_{CLK}$ upon being downconverted to $f_{IF}=f_{notch}=|f_{RF}-f_{LO}|$. The combination of the notch and comb filter at the IF bandwidth with k_1 = -1/2 , k_2 = 255/256, f_{CLK}=45.3MHz, and *K*=15/16 is shown in Fig. 6. Note the peaks at the harmonics of f_{rev}=52.85MHz/588. Delay T_d is not used here.

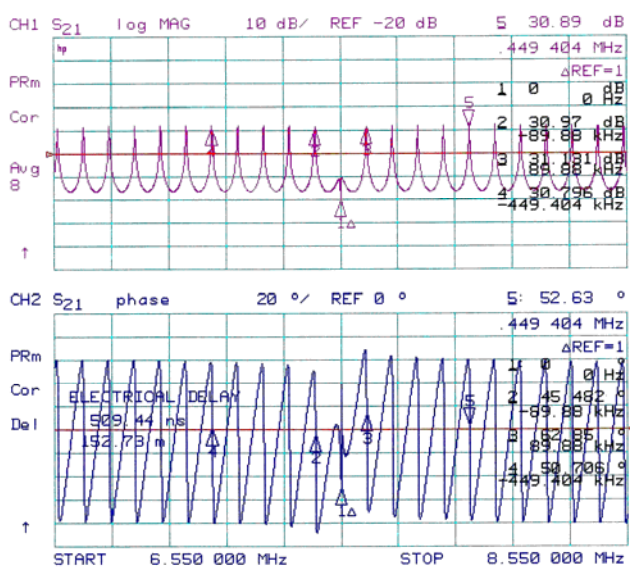

Figure 6: Comb+Notch filter measurement k_1=-1/2, k_2=255/256, f_{CLK}=45.3MHz, f_{IF}=7.55MHz, *K*=15/16.

Hilbert Pair Filters–Digital Quadrature Splitter

The Hilbert pair filters are based upon a digital Hilbert transform filter design technique [6]. The coefficients of a LPF finite impulse response (FIR) prototype filter are modulated with a complex exponential to invoke the frequency shift property of the Fourier transform: $c(n) = h(n) \cdot e^{j2\pi \frac{f_{\mathrm{mod}}}{f_{CLK}} n} \Leftrightarrow C(f) = H(f - f_{\mathrm{mod}})$ where h(n) are the LPF filter coefficients, $H(f)$ the LPF frequency response. This forms a complex BPF with complex coefficients *c(n)* whose response $C(-f) \cong 0$ for f in the filter passband, thus approximating an analytic signal in the passband. The calculated *C(f)* response formed from a 30th order LPF prototype is shown in Fig. 7. If the real and imaginary coefficients of the complex BPF are used to form two separate real FIR BPFs, their outputs form a Hilbert pair over the passband. Minimal passband ripple is found for $f_{\mathrm{mod}} / f_{CLK} = 1/4$.

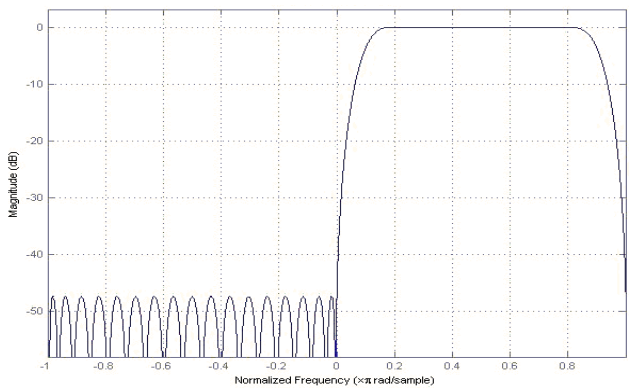

Figure 7: Calculated complex BPF magnitude response.

A response of a phase shifter at the IF bandwidth formed from the Hilbert pair plus phase rotation is shown in Fig. 8. Note the relatively constant phase shift across the passband. The magnitude and phase roll off above 10MHz includes the 17MHz LPFs used for anti-aliasing at the ADC inputs and reconstruction at the DAC outputs.

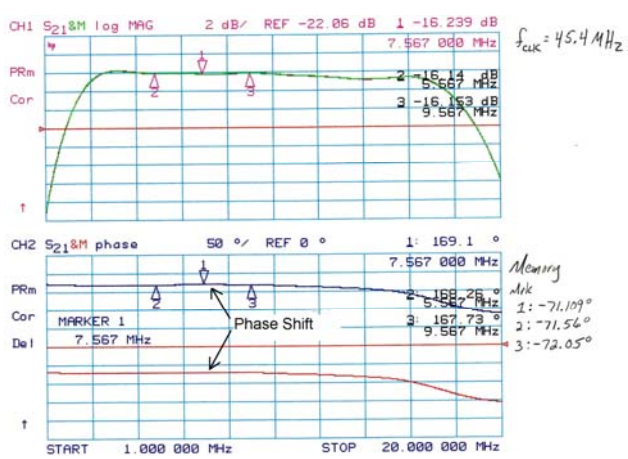

Figure 8: Magnitude (top) and phase (bottom) of the Hilbert pair + rotation at two rotations 240deg apart (blue and red phase traces).

Near One-Turn Delay

Since $e^{-j2\pi f \cdot T_{rev}} = 1 \; \forall \; f : f = m \cdot f_{rev}$ with m an integer and $T_{rev}=1/f_{rev}$, the purpose of the 'one-turn-delay' design [1] is to extend the unavoidable loop delay such that the comb open loop phase (neglecting the negative feedback 180deg phase flip) equals zero at all the revolution harmonics where the loop has gain, thereby maintaining loop stability. Due to the transcendental terms in Eq. 1 arising from the time delays, stability analysis is more easily accomplished graphically via Nyquist plots.

It is phase, not group-delay, that is the important parameter in designing for the comb loop stability. Since group-delay is $-d\phi/d\omega$, it neglects a constant phase offset, thus using group-delay equalization alone does not solve the stability. Instead, phase equalization should be the objective. The largest deviation from linear phase in the comb loop is due to the effective cavity impedance whose phase response is $\phi_Z \cong \tan^{-1}(2Q_{eff}(\omega_o - \omega)/\omega)$ with $Q_{eff} = Q_{cav}/(1+\beta_{Direct})$ the reduced cavity quality factor Q_{cav} due to the Direct RF FB loop gain β_{Direct}.

If a group-delay equalizer is designed as in [4], one should also be sure to include a phase offset to complete the phase equalization. An alternative solution to the phase equalization is to simply reduce and balance the deviation from linear phase due to the effective cavity impedance across the useable comb loop bandwidth by using a delay $T_d = T_{rev} - \tau_{sys} - \alpha\tau_Z(f)\big|_{f=f_{RF}}$ where τ_{sys} is the unavoidable system cabling delay, $\tau_Z(f)$ is the effective cavity impedance group delay, and α is an adjustable parameter. Since T_d is implemented digitally (see Eq. 2), α is also chosen to force T_d equal to an integer number of clock periods. Finally, a phase rotation multiplier, $e^{j\phi_{offset}}$, is added to Eq. 1 to center the phase response about zero across the useable loop bandwidth. The result of such a design is shown in Figs. 9-11. Note the phase margin increase and closed loop impedance improvements for $T_d = T_{rev} - \tau_{sys} - \alpha\tau_Z$ for α > 0.

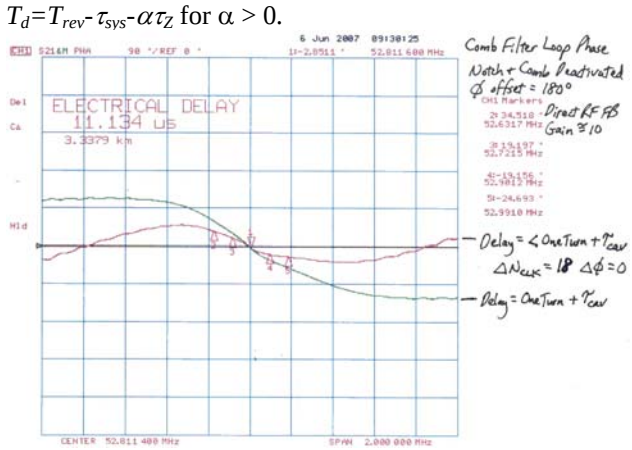

Figure 9: Comb feedback open loop phase response at the RF bandwidth with comb and notch filter deactivated [4] with β_{Direct} ~=10 and appropriate ϕ_{offset} settings: (green) $T_d = T_{rev} - \tau_{sys}$ [α=0] (red) $T_d = T_{rev} - \tau_{sys} - \alpha\tau_Z$ [α > 0].

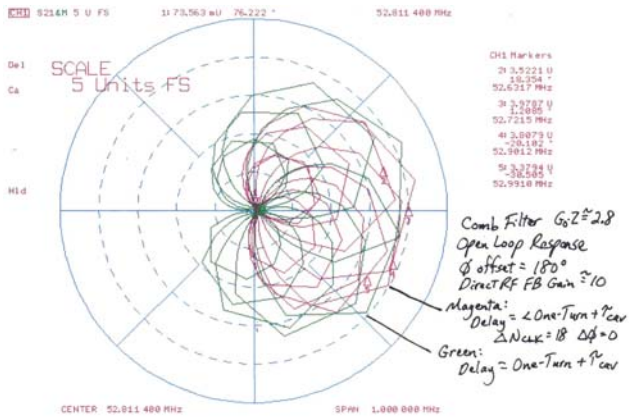

Figure 10: Polar plot of open loop response (effective Nyquist plot) with notch and comb filter active with β_{Direct} ~=10 and appropriate ϕ_{offset} settings: (green) $T_d = T_{rev} - \tau_{sys}$ [α=0] (red) $T_d = T_{rev} - \tau_{sys} - \alpha\tau_Z$ [α > 0].

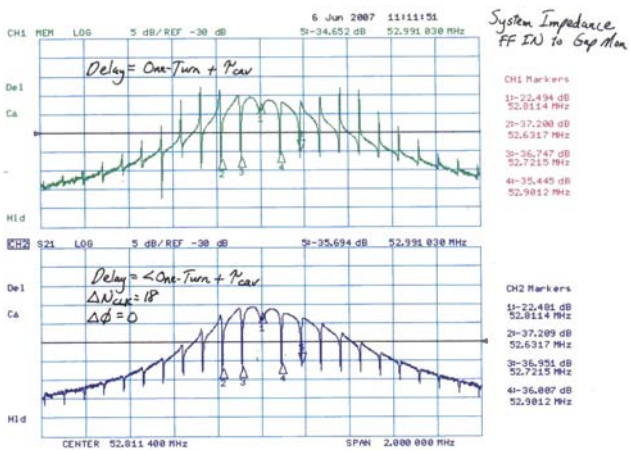

Figure 11: Closed loop response (effective impedance to beam) with β_{Direct} ~=10 and appropriate ϕ_{offset} settings: (top) $T_d = T_{rev} - \tau_{sys}$ [α=0] (bottom) $T_d = T_{rev} - \tau_{sys} - \alpha\tau_Z$ [α > 0].

In Figs. 9-11, the normalized comb filter loop gain was ~2.8 (as defined at resonance with no Direct RF FB). If one could correct exactly for the cavity phase response, much higher gains can be realized, as the Nyquist plot would look like a set of parallel resonators for which the stability condition of [1] would apply to each. Since the gain margin is determined by the point at which the loop gain is highest, the Direct RF FB allows for increased comb gain by flattening the gain near f_{RF} (as seen by the comb loop which is effectively outside the Direct RF FB loop). The notch filter essentially eliminates the comb loop gain at f_{RF} and reduces it immediately near f_{RF}. Additionally, the Direct RF FB increases the phase margin by reducing the linear phase deviation of the effective cavity impedance and the effective phase shifts caused by cavity detuning. Software tools were used to explore many aspects of loop stability and were proven to be invaluable in predicting performance for various feedback loop parameters.

OPERATIONAL STATION RESULTS WITH BEAM

The prototype comb filter feedback loop was added at injection to an operational MI RF station during a typical cycle. Figures 12 and 13 compare vector signal analyzer (VSA) measurements of the cavity gap monitor with and without the comb feedback. The VSA measurements were taken with a beam filling pattern of: 1/7 filled with a double intensity (~1E13) slip-stacked batch and 1/7 filled with a single intensity (~0.5E13) batch with a 1/14 gap between the batches. The comb filter offered an additional ~12dB reduction at the first few beam loading revolution harmonics for a total of ~30dB reduction when working in conjunction with the existing feed-forward (FF) system.

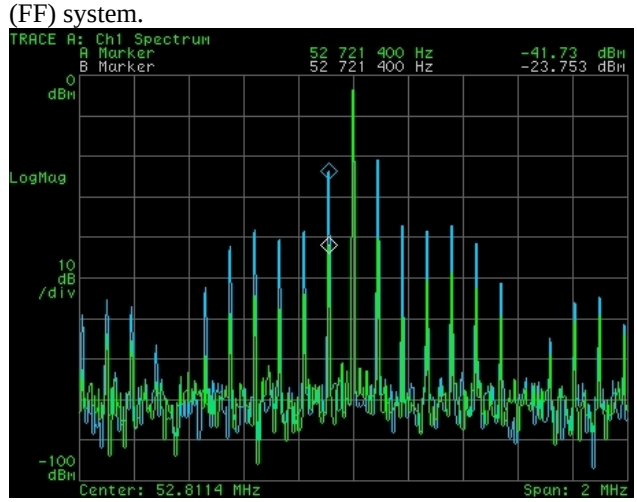

Figure 12: VSA measurement of cavity gap voltage with Comb Loop OFF, Direct RF FB active and (blue) – FF OFF, Comb OFF, (green) – FF ON, Comb OFF.

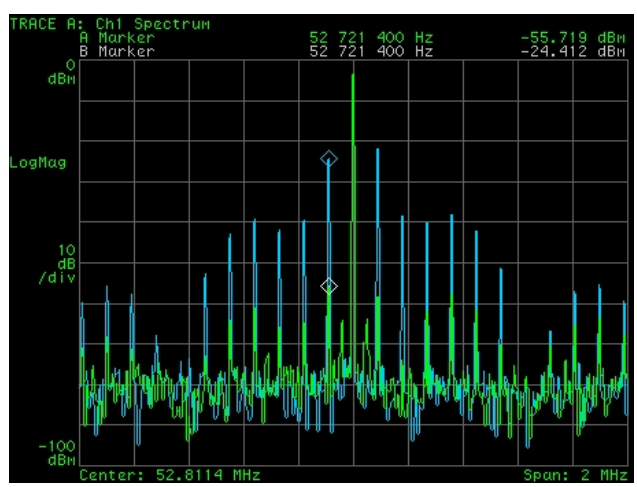

Figure 13: VSA measurement of cavity gap voltage with Direct RF FB active and (blue) – FF OFF, Comb OFF, (green) – FF ON, Comb ON. The spurs between f_{RF} and the first revolution harmonics were found to be associated with FF, not the comb loop.

Figures 14 and 15 compare the time domain results with and without the comb feedback. Both are images of the cavity gap voltage envelope magnitude and phase taken later in the cycle when the ring has the following filling pattern: 1/7 filled with a double intensity (~1E13) slip-stacked batch and 3/7 filled with single intensity (~0.5E13) batches with a 1/14 gap between the dobule intensity batch and the single intensity group. The four batches within the revolution period of ~11usec is clearly seen. Note the system time constant reduction and approximate factor of three reduction in the beam induced amplitude and phase modulation with the addition of the comb feedback.

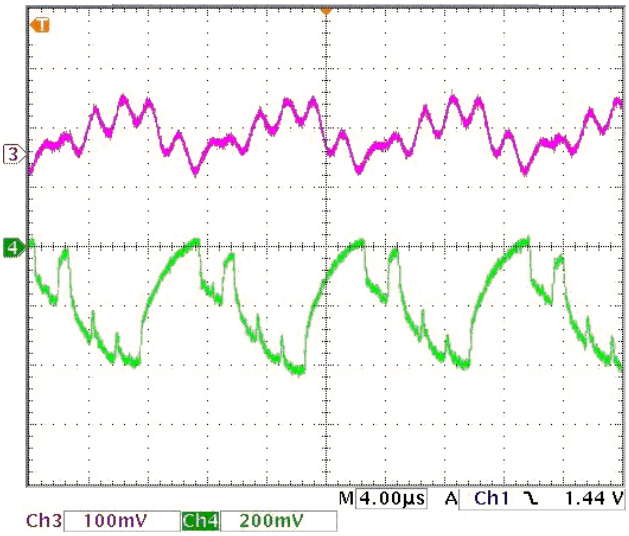

Figure 14: Cavity voltage envelope (top) magnitude ~5kV/div and offset on, (bottom) phase 1.8deg/div with FF ONLY.

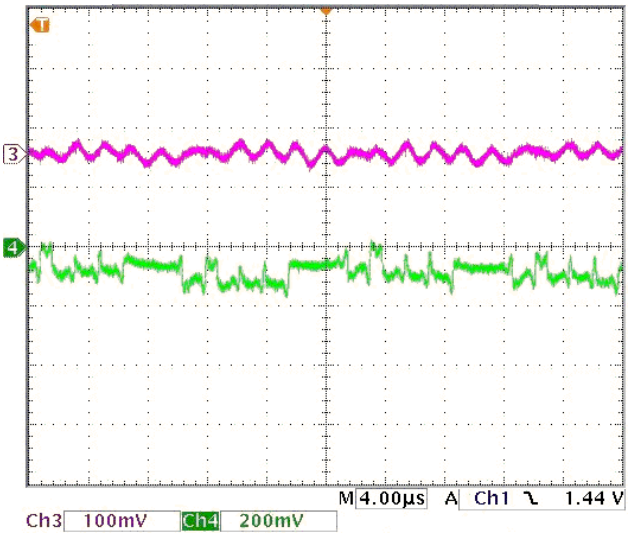

Figure 15: Cavity voltage envelope (top) magnitude ~5kV/div and offset on, (bottom) phase 1.8deg/div with FF and Comb Feedback both ON.

CONCLUSION

A prototype comb feedback loop was designed and tested on an operational MI RF station at a single RF frequency. Included was a practical notch filter application, a digital quadrature splitter application, and a simple solution to the phase equalization problem. The combination of Direct RF Feedback, a notch filter, and phase equalization allows for increased comb loop gain and stability. By complementing the existing feed-forward system, the comb loop offers additional beam loading reduction along with the advantages of a feedback system. The combination of the two systems allows them to complement each other. The prototype test is a proof of principle giving confidence in the design. Future work includes designing the loop to work across the operational RF frequency sweep and optimizing loop gain.

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