Quadrupole-mode Maeasurements and their applications

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- The quadrupole-mode transfer function (QTF) is a powerful non-destructive tool to measure properties of dynamical systems. The QTF can be used to measure the betatron tunes and the rms beam emittances with a beam position monitor (BPM) systems. The QTF can also be used to compensate the optical mismatch during the beam injection process.

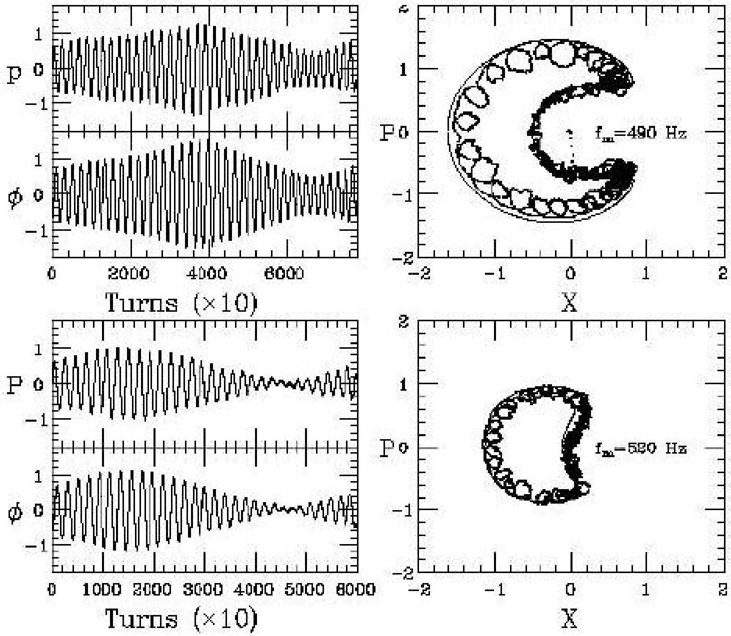
- Introduction
- Quadrupole-mode Transfer Function, what is it?
- Applications
-     Emittance and Tune Measurements
-     Mismatch compensation
-     Overcoming Intrinsic Spin Resonances
- Conclusion

- **Applications of parametric resonances of dynamical systems in beams**

- ❑ **Characterizing chaos and properties of many dynamical systems: QI-dynamical system, space charge dominated beams, etc.**
- ❑ **Parametric resonances in the longitudinal phase-space induced by rf cavity voltage and phase modulations can be employed to actively compensate the synchro-betatron coupling resonances, to create a bounded chaotic region in the longitudinal phase-space for a controlled bunch dilution.**
- ❑ **Voltage modulation at the second synchrotron sideband has also been applied to alleviate the coupled bunch instability driven by the parasitic modes, and to manipulate bunch shape for bunch length compression.**
- ❑ **The idea of bunch manipulation has recently been extended to the transverse phase-space, where the coherent dipole mode excitation driven by a transverse rf dipole field has been successfully applied to overcome intrinsic spin resonances at the AGS. The rf dipole, excited adiabatically, induces coherent betatron oscillations, which provides a method to measure the betatron tune without suffering emittance dilution.**
- ❑ **With advanced data analysis techniques, the dipole-mode transfer function can be used to reveal hidden dynamical variables in the orbit response matrix method, and the model independent analysis method. Both techniques have been successfully implemented in improving the performance of high intensity accelerators.**

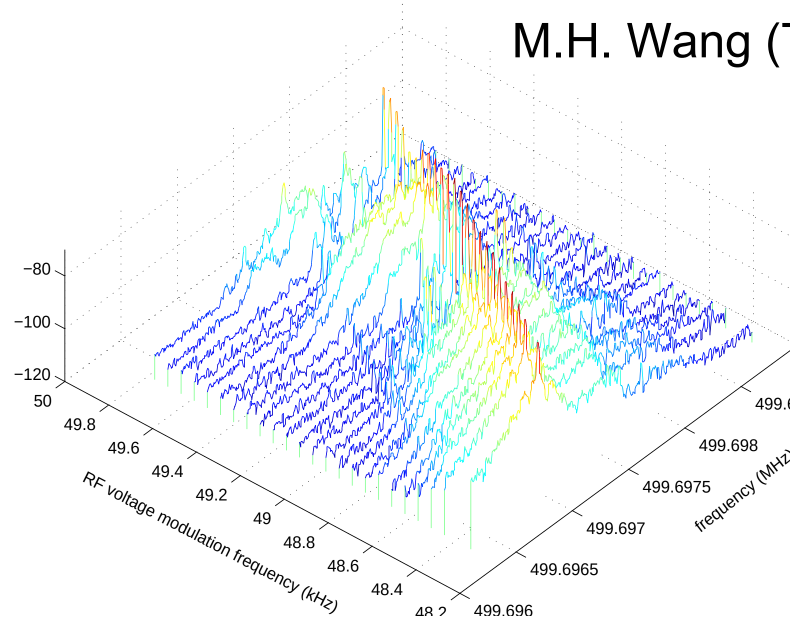
- **How about the quadrupole-mode transfer function?**

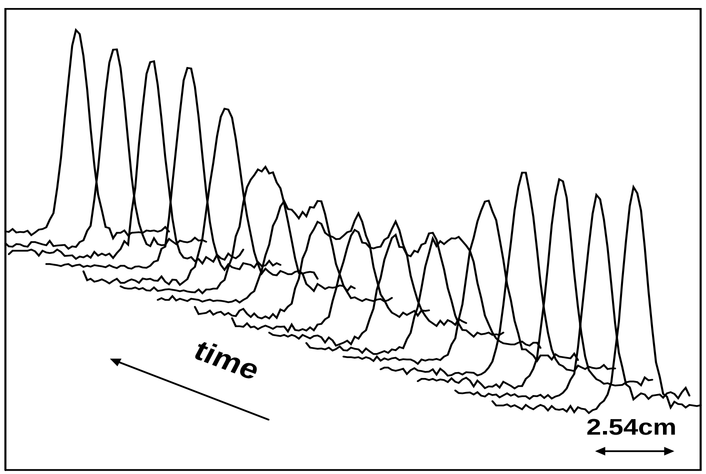
- ➢ **A fast field-changing quadrupole had been used to produce betatron tune jump for overcoming intrinsic spin resonances and for studying a strong betatron resonance.**
- ➢ **A harmonic tune modulation can be used to study the effect of enhanced-diffusion process at a betatron resonance.**
- ➢ **Other Applications?**



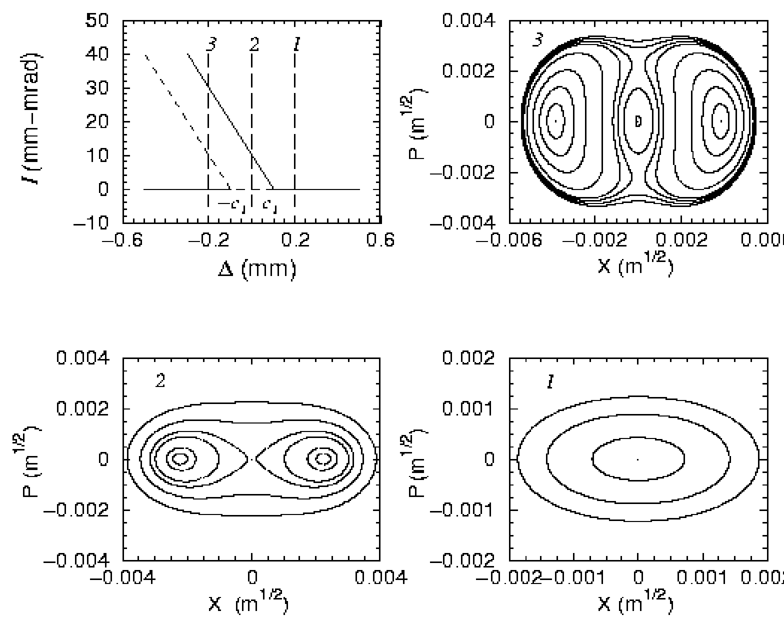
CE22 (IUCF)

M.H. Wang (TLS)





M. Bai (AGS)



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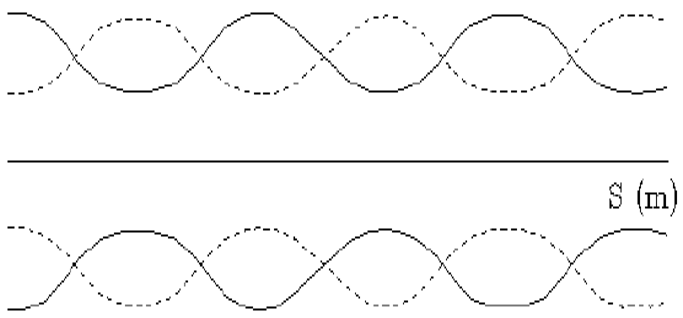
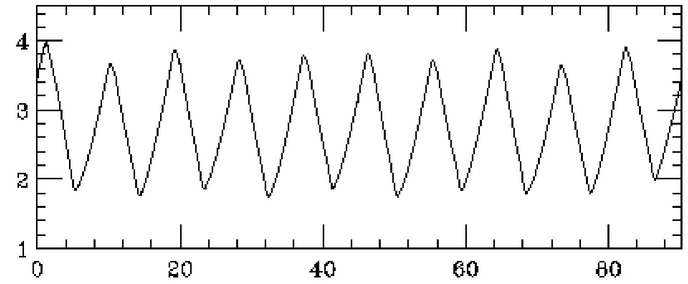
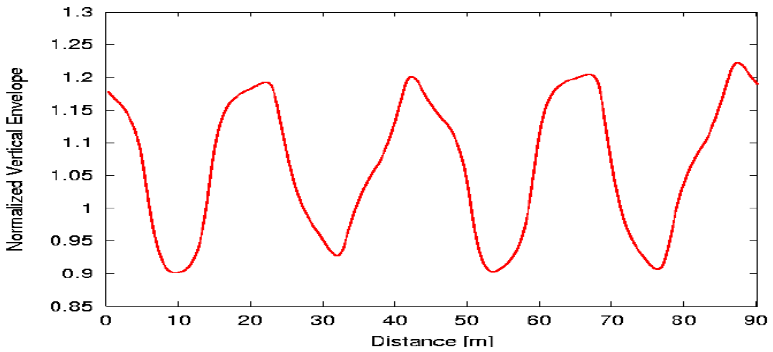
The envelope oscillations:

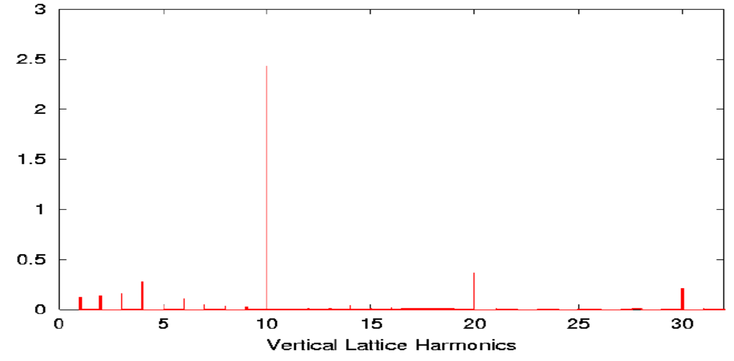
The envelope of particle beams in accelerators is determined by the (square root) betatron amplitude function and the beam emittance as shown schematically in the following graphs:





For space charge dominated beams, the envelope oscillations has another complication. The following graph shows the additional envelope oscillations for the PSR, obtained from a PIC space charge simulation code [S. Cousineau et al].





So, what is the point?

- **If we compare the envelope oscillation of the regular betatron amplitude with that of a uniform focusing accelerator, the beam envelope modulation can drive particle-envelope interaction due to the space charge force. However, the oscillation frequency of the envelope differs substantially from twice the betatron frequency of particles. The effect is not very important!**
- **Now, we consider a "not so perfect" lattice with space charge. As the envelope tune is pushed downward near an integer, where the lattice perturbation exists. The envelope will be forced to oscillate at the forced oscillation frequency. This is exactly what we see from the PIC code calculation at n=4 for the PSR lattice in the vertical plane.**
- **This large envelope oscillation at n=4 will induce emittance growth by driving particle resonantly outward.**

$$H = \frac{1}{4\pi} p^2 + \frac{1}{4\pi} k_y(\theta) y^2 - \frac{K}{4\pi R^2} y^2 \Theta(R - |y|) - \frac{K}{4\pi}\left(1 + 2\ln\frac{y}{R}\right)\Theta(|y| - R)$$

$$K = \frac{2Nr_0L}{\beta^2\gamma^3\varepsilon}$$

Quadrupole-mode measurements

$$q_2 = \frac{L+R-U-D}{\Sigma} = \frac{2}{a^2}\left(\langle x^2\rangle + \langle x\rangle^2 - \langle z^2\rangle - \langle z\rangle^2\right)$$

- Profile monitors – measure the deviation from the profile in a regular lattice for the space charge dominated beams.
- One needs also BPM to model betatron motion using dipole-mode transfer function methods. (Model independent analysis)
- What can one learn from the quadrupole mode measurements using a quadrupole pickup?
  (1) the deviation from the regular betatron oscillation gives the perturbation strength; (2) detailed data analysis can be used to confirm accelerator modeling; (3) it provide information on machine and beam parameters; (4) it provides active compensation of optical mismatch, etc.

Transfer Function

- The transfer function is defined as the ratio of response to a given known excitation. This transfer function can be measured from a network analyzer, or spectrum analyzer with input signal calibration.
- Transfer function method has been widely applied in measurements of longitudinal impedance, beam cooling properties, "model independent analysis", etc.
- However, the quadrupole-mode transfer function (qtf) has not been considered. To measure qtf, an rf quadrupole is used as a kicker, and the response can be measured by quadrupole pick-up. With a single harmonic excitation, the quadrupole mode function is

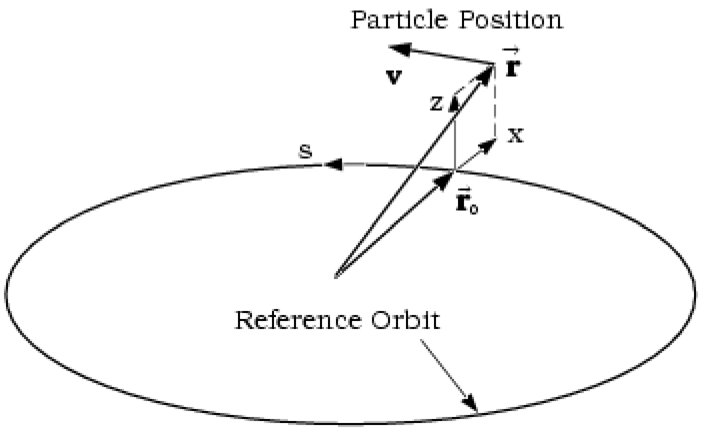
$$\langle x^2 \rangle = \tfrac{1}{2}(\langle X^2 \rangle + \langle P^2 \rangle) + \tfrac{1}{2}(\langle X^2 \rangle - \langle P^2 \rangle)\sin(\nu_m \theta)$$

$$q_2 = \tfrac{1}{a^2}(b_0 + b_1 \sin(\nu_m \theta))$$

The Hamiltonian of particle motion in accelerators is governed by

$$H = \tfrac{1}{2} y'^2 + \tfrac{1}{2} K_y(s) y^2 + \tfrac{1}{2} K_{rf}(s) y^2 \cos(\omega_m t + \vartheta_0)$$

Transforming into the action angle variables, we obtain the Hamiltonian as



$$H(J_y, \psi_y) = \nu_y J_y + C_1 J_y \cos(2\phi_y - n\theta - \nu_m \theta + \chi)$$

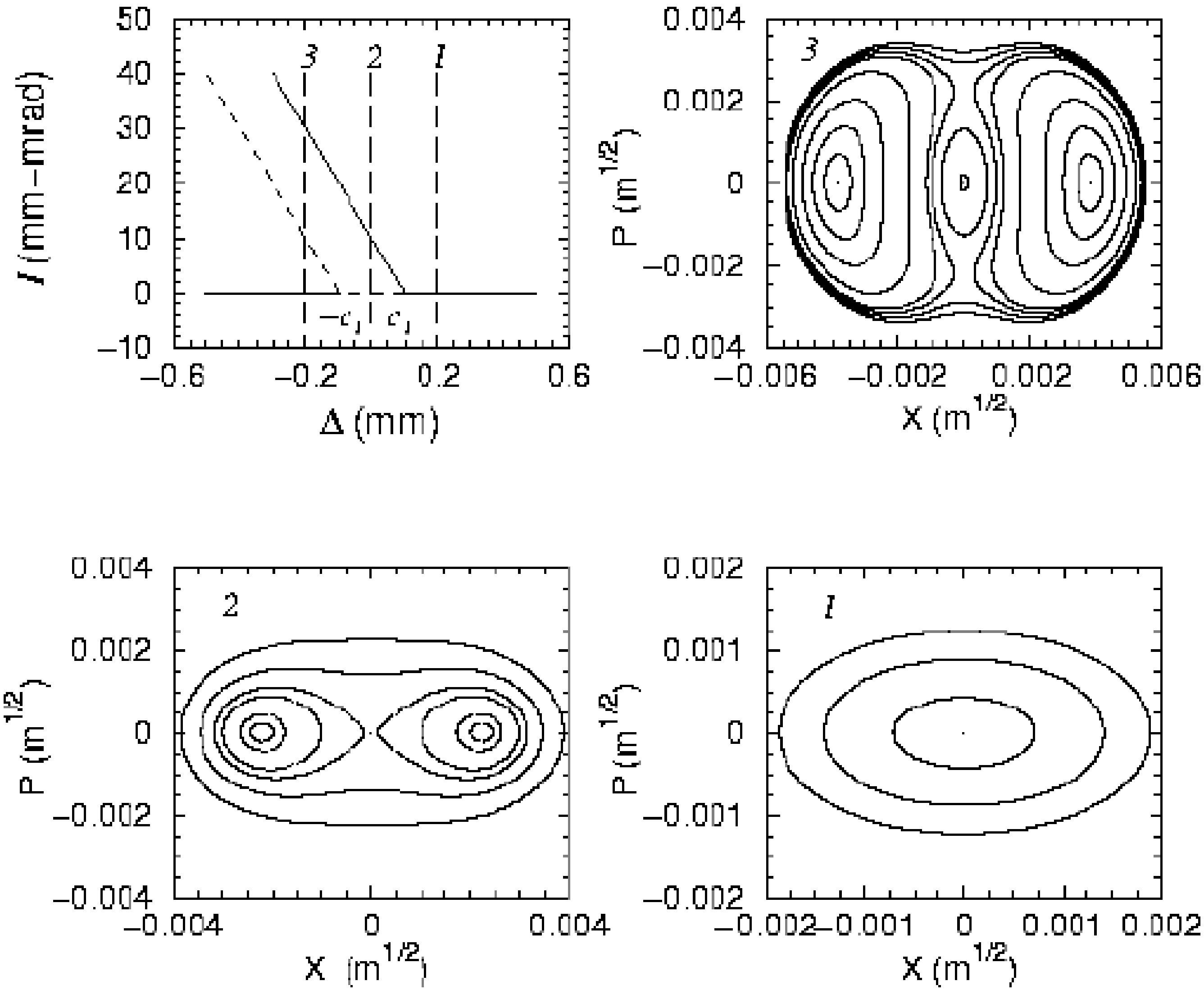
$$I = J_y, \psi = \phi_y - \tfrac{1}{2} n\theta - \tfrac{1}{2}\nu_m \theta + \tfrac{1}{2}\chi$$

$$H = \delta I + C_1 I \cos(2\psi) + [\tfrac{1}{2}\alpha I^2]$$

$$\delta = |\nu_y - \tfrac{1}{2} n| - \tfrac{1}{2}\nu_m$$

$$C_1 = \left| \oint \frac{\beta(s)}{8\pi} K_{rf}(s) e^{-jns/R \pm j(2\mu(s) - 2\nu s/R \pm \vartheta(s))} ds \right|$$

$$\Delta = \delta / \alpha$$



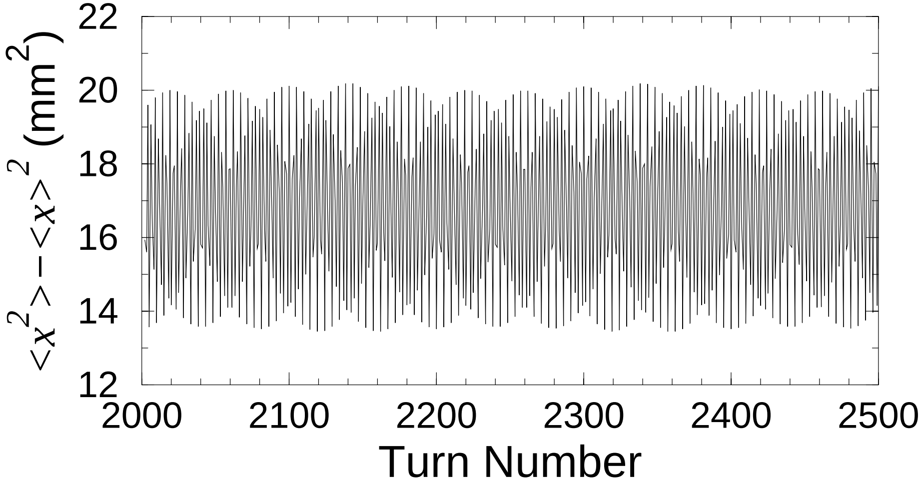
Applications of quadrupole-mode transfer function

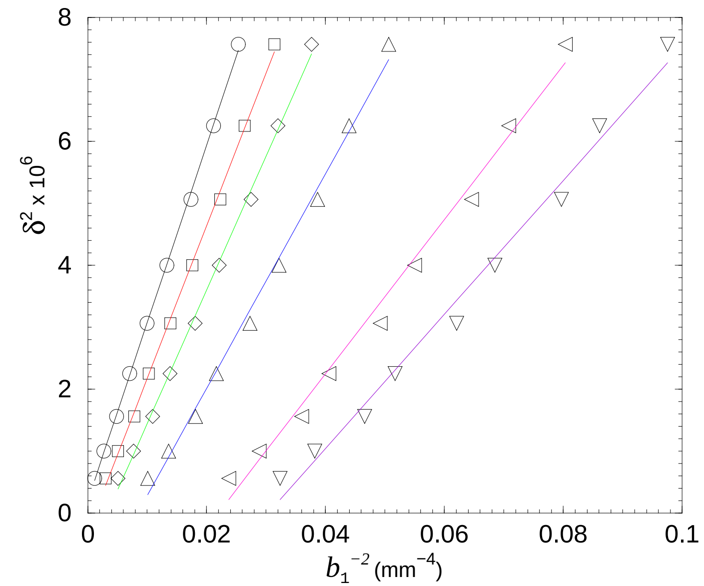
- Measurement of the betatron tune and rms emittance without emittance dilution. Procedure: rf quadrupole field strength is adiabatically increased, then the rf frequency is adiabatically changed to near the betatron sideband frequency, the response of beam signal measured, and restore to the original operation condition.
- Injection mismatch compensation.
- Impedance measurement?

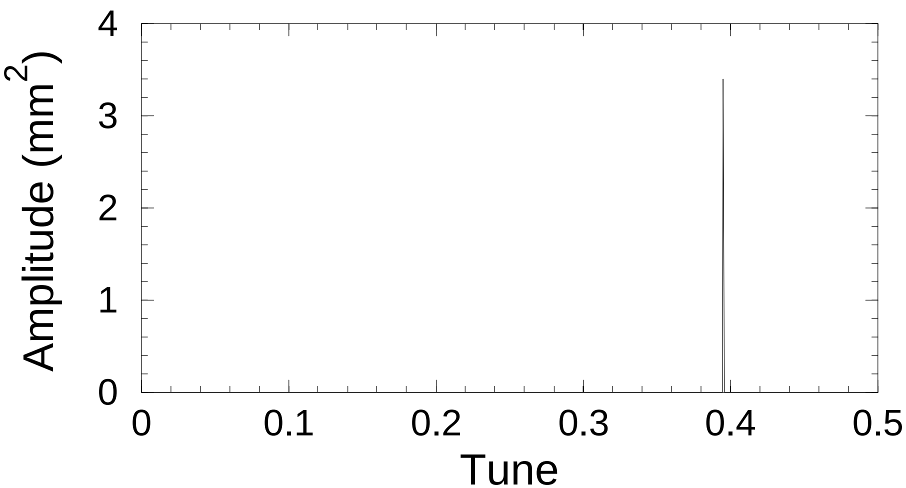
- Bunch compression for longitudinal quadrupole mode excitation.

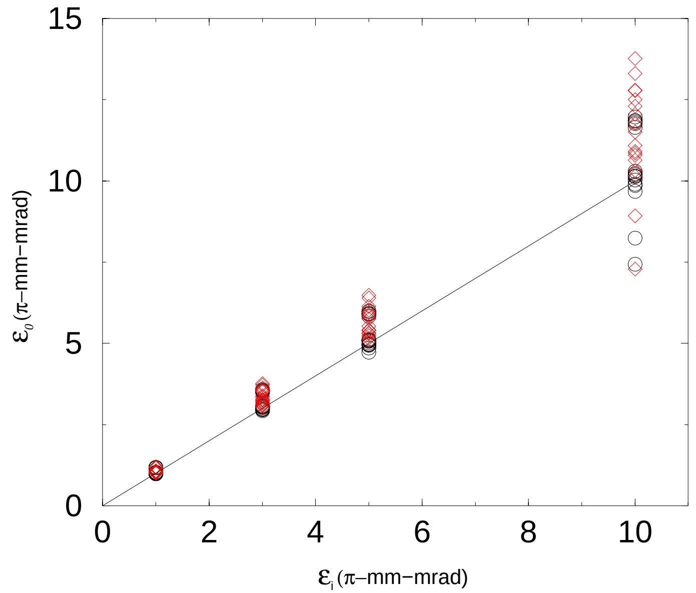
$$\delta^2 = C_1^{\,2} + (4C_1^{\,2}\beta_x^{\,2}\varepsilon^2)\frac{1}{b_1^{\,2}}$$

Numerical Example for rms emittance measurements









Summary

- Quadrupole-mode measurements and the half-integer stopband corrections are important for space charge dominated beams.
- Quadrupole-mode transfer function (QTF) can be used to obtain the rms emittance of the beam bunch without emittance dilution. Need experimental developments for this techniques. QTF can also be used to measure the betatron tune.
- QTF can also be used to compensate optical mismatch in the injection process.
- Hardware requirements: rf quadrupoles, quadrupole-mode monitors, etc.