Low-β BPM Response using Log Ratio and Difference over Sum Processing

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The four plots on the following pages show the effect of low-β response for an 805-MHz, 1.5-cm radius, 60º-electrode BPM. This is approximately the BPM design used in the SNS DTL drift tubes. The six solid (colored) lines represent 2.5, 5, 10, 20, 50, and 1000 MeV protons.

There are three types of processing used for normalized BPM position ouput, difference over sum(DOS), log ratio, and amplitude to phase conversion (AMPM) We examine the low beta response of DOS and log ratio. The present design for the SNS BPM system uses difference over sum processing.

$$DOS = \frac{R-L}{R+L}$$

$$Log\ \ ratio = Ln\left(R/L\right) = 2\cdot\tanh^{-1}\left[\frac{R-L}{R+L}\right]$$

The four Figures are:
Fig 1: vertical offset y = 0 cm; log ratio processing
Fig 2: vertical offset y = 0.5 cm, log ratio processing
Fig 3: vertical offset y = 0 cm, difference-over-sum processing
Fig 4: vertical offset y = 0.5 cm; difference over sum processing

In all four plots, a black dotted line represents the response for β=1 and y=0 (using the 2D Laplace equation solution). In all plots, the output values are normalized so that the initial slope (for x<<b) and for y=0 is 1.

The 3D Laplace equation solution for the low-beta case is given by

$$R = \left\{\frac{I_0(gr)}{I_0(gb)} + \frac{4}{\varphi}\sum_{n=1}^{\infty}\frac{1}{n}\frac{I_n(gr)}{I_n(gb)}\cos(n\theta)\sin\left[n(\varphi/2)\right]\right\}$$

$$L = \left\{\frac{I_0(gr)}{I_0(gb)} + \frac{4}{\varphi}\sum_{n=1}^{\infty}\frac{1}{n}\frac{I_n(gr)}{I_n(gb)}\cos(n\theta)\sin\left[n(\pi+\varphi/2)\right]\right\}$$

Where the beam is at (r,θ), the electrode angular width is φ, and I_m(arg) is the modified Bessel function of order m, and

$$g = \frac{\omega}{\beta\gamma c}$$

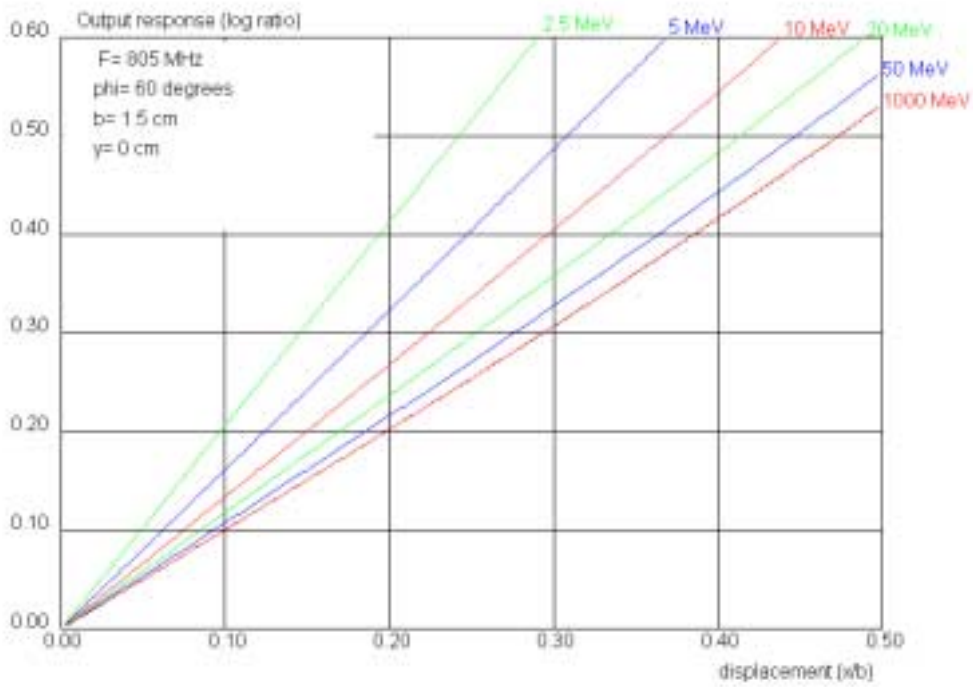

Figure 1.Log ratio, y=0 cm

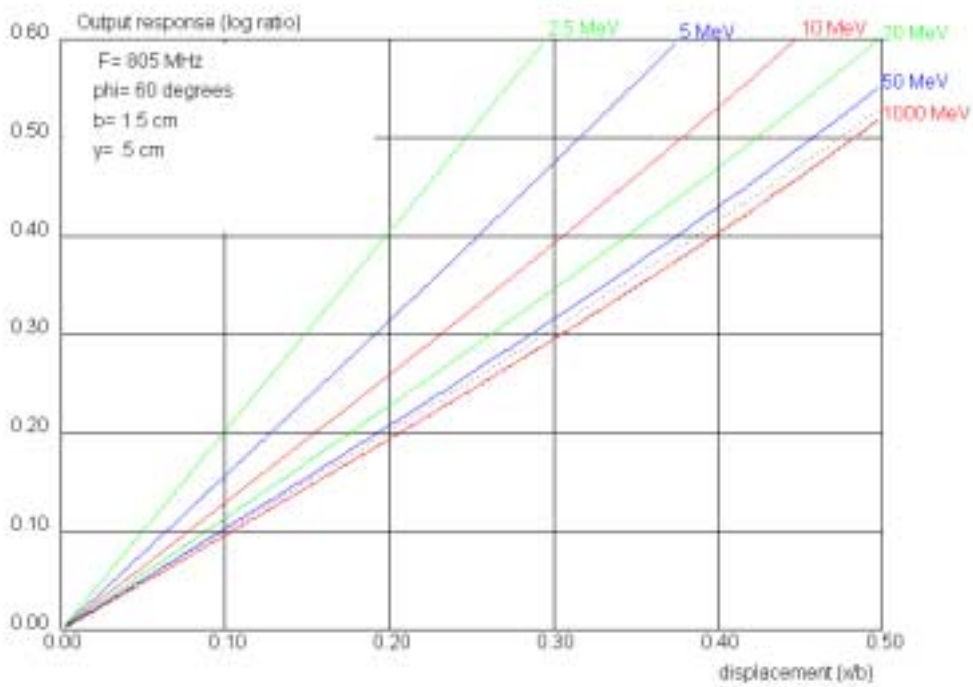

Figure 2. Log ratio, y=0.5 cm. Note slight offset from y=0 (dotted line).

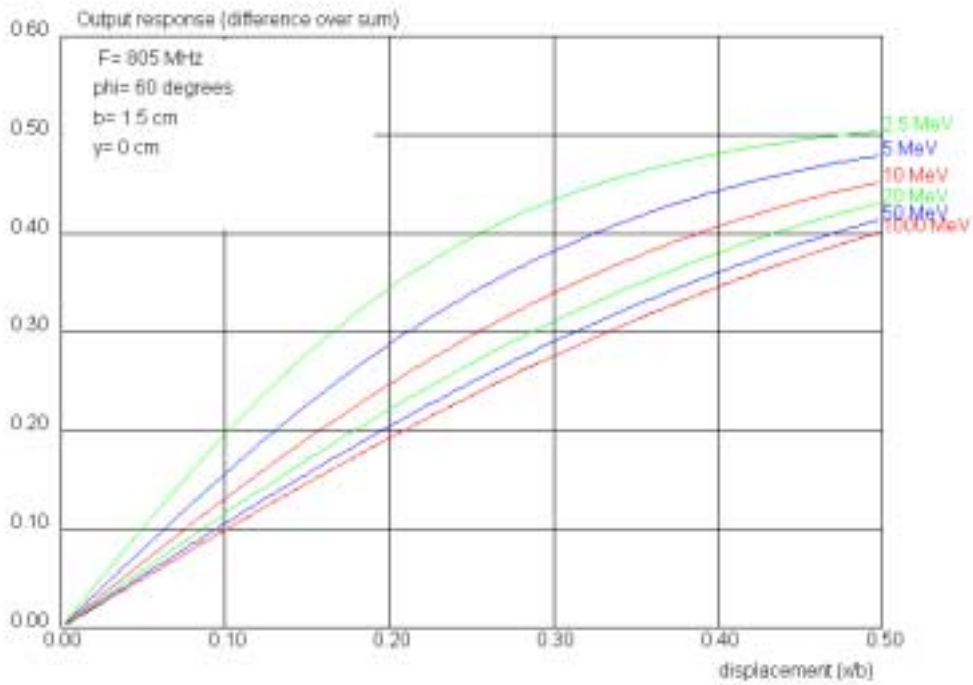

Figure 3. Difference over sum, y=0 cm

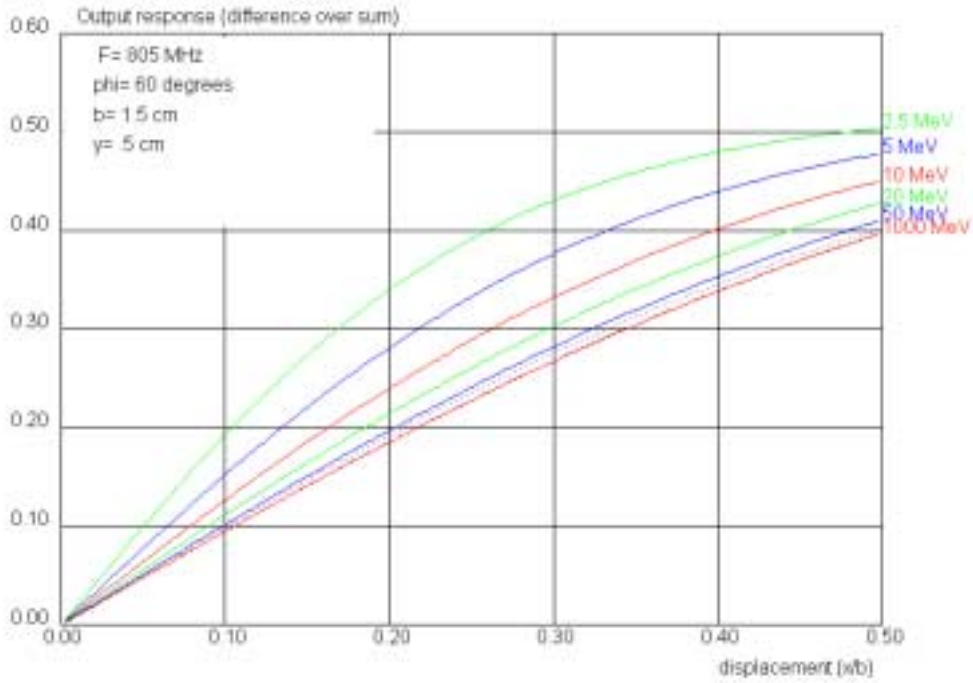

Figure 4. Difference over sum, y=0.5 cm. Note slight offset from y=0 (dotted line).