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CALCULATION OF THE NEUTRALIZATION DEGREE IN H- BEAM DUE TO INTERACTION WITH LASER BEAM.

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Introduction.

The purpose of this paper is to calculate yield of photo-neutralization reaction for H-beam intersecting stream of photons from a laser. Cross-section of the reaction is well studied elsewhere and proper interpolation formula e.g. from [1] can be used for numeric calculations (suitable interpolation formula is given in the Appendix). In order to find reaction yield when cross-section is known one has to calculate geometric factor of the collision sometimes referred as luminosity [2]. It can be done analytically for some simple shapes of the colliding beams. Formulae for infinite cylindrical beams with Gaussian density distribution can be found in [3] for example. This formula is not applicable in case of short bunches which may be of interest for certain applications such as longitudinal distribution measurement using a mode-locked laser. In this paper analytical formula for collision of the 3-D Gaussian beams of arbitrary r.m.s. sizes is derived. Saturation of the yield due to depletion of the ion beam in the interaction region is not taken into account which is justified for power levels of the commercial mode locked lasers. When the saturation effect is significant numerical integration should be used.

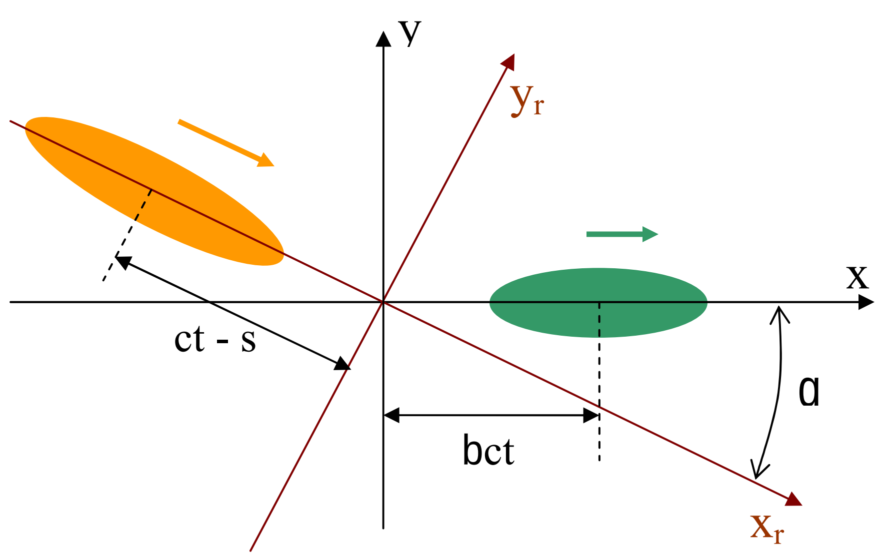

Figure 1. Intersection layout

Photo-neutralization yield.

The rate of the photo-neutralization reaction can be expressed as

$$R = c(1-\beta\cos\theta)\cdot G\cdot\Sigma_0,$$

where c is speed of light, $\beta = \frac{v}{c}$ is particle velocity, Σ_0 is invariant cross-section (measured in beam frame of reference) and G is factor depending on geometric configuration of the collision, which can be expressed as

$$G = \iiint n_l n_b dV,$$

where n_l is photon density in the laser beam, n_b is particle density in the ion beam. If density depends on time explicitly as in case of beams of finite length, then reaction rate depends on time and the total yield can be found by integration over total time of the interaction:

$$Y = c(1-\beta\cos\theta)\Sigma_0\int dt\iiint dV n_l(t)n_b(t).$$

Let's consider 3-D laser and ion beams intersecting at angle θ as shown in fig.1. Then

$$n_b = n_{b0}\cdot e^{-\frac{(x-\beta ct)^2}{2\sigma_{bs}^2}}\cdot e^{-\frac{y^2}{2\sigma_{by}^2}}\cdot e^{-\frac{z^2}{2\sigma_{bz}^2}},$$

where $n_{b0} = \frac{N_b}{(2\pi)^{3/2}\sigma_{bs}\sigma_{by}\sigma_{bz}}$ is ion density, $\sigma_{bs},\sigma_{by},\sigma_{bz}$ are r.m.s. beam sizes along s, y, z respectively. $N_b = \frac{I_b}{e\cdot f_{RF}}$ is number of particles in the ion beam expressed through beam current I_b, bunch repetition frequency f_{RF} and electron charge e.

$$n_l = n_{l0}\cdot e^{-\frac{(x_r-ct-s)^2}{2\sigma_{ls}^2}}\cdot e^{-\frac{y_r^2}{2\sigma_{ly}^2}}\cdot e^{-\frac{(z_r-d)^2}{2\sigma_{lz}^2}}$$

where $\begin{aligned} x_r &= x\cos\theta - y\sin\theta, \\ y_r &= x\sin\theta + y\cos\theta, \\ z_r &= z. \end{aligned}$ are coordinates in the frame rotated by angle θ relative to the laboratory frame as shown in fig.1, d is vertical shift of laser beam axis relative to ion beam axis, s is delay of the laser beam arrival to the collision point relative to the ion beam (when $s=0$ centers of both beams meet at the center of coordinate system). $n_{l0} = \frac{N_l}{(2\pi)^{3/2}\sigma_{ls}\sigma_{ly}\sigma_{lz}}$ is photon density, $N_l = \frac{W_l\cdot\lambda}{h\cdot c}$ is number of photons in the laser beam expressed through light pulse energy W_l, laser wavelength and Plank constant h.

Values for λ and W_l should be taken in the laboratory frame of reference. Substituting density into the expression for the yield we have: $Y = A \cdot F \cdot F_z$, where:

$$A = \frac{I_b W_l \lambda (1 - \beta \cos\theta) \Sigma_0}{h e f_{RF} (2\pi)^3 \sigma_{bs} \sigma_{by} \sigma_{bz} \sigma_{ls} \sigma_{ly} \sigma_{lz}},$$

$$F = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{(x-\beta ct)^2}{2\sigma_{bs}^2} - \frac{y^2}{2\sigma_{by}^2} - \frac{(x\cos\theta - y\sin\theta - ct - s)^2}{2\sigma_{ls}^2} - \frac{(x\sin\theta + y\cos\theta)^2}{2\sigma_{ly}^2}} dt dx dy,$$

$$F_z = \int_{-\infty}^{\infty} e^{-\frac{z^2}{2\sigma_{bz}^2} - \frac{(z-d)^2}{2\sigma_{lz}^2}} dz.$$

Calculation of F_z is straightforward and gives:

$$F_z = \sqrt{2\pi} \frac{\sigma_{bz}\sigma_{lz}}{\sqrt{\sigma_{bz}^2 + \sigma_{lz}^2}} e^{\frac{-d^2}{2(\sigma_{bz}^2 + \sigma_{lz}^2)}}.$$

Integration in F can be done in closed form but it takes significant algebraic manipulation, which is omitted here due to large volume of calculation. The resulting expression is

$$F = \frac{2\sqrt{2}\pi^{3/2}}{c} \frac{\sigma_{bs}\sigma_{by}\sigma_{ls}\sigma_{ly}}{\sigma_{eff} \beta \sin\theta} \cdot e^{-\frac{s^2}{2\sigma_{eff}^2}}$$

where

$$\sigma_{eff} = \sqrt{\frac{\sigma_{bs}^2 \sin^2\theta + \sigma_{by}^2 (\beta - \cos\theta)^2 + \sigma_{ls}^2 \beta^2 \sin^2\theta + \sigma_{ly}^2 (1 - \beta\cos\theta)^2}{\beta^2 \sin^2\theta}}.$$

Final expression for the total yield is

$$Y = \frac{1}{2\pi hce}\frac{I_b \lambda W_l}{f_{RF}}\frac{(1-\beta\cos\theta)}{\beta\sin\theta}\frac{\Sigma_0}{\sigma_{eff}\sqrt{\sigma_{bz}^2+\sigma_{lz}^2}}e^{-\frac{s^2}{2\sigma_{eff}^2}-\frac{d^2}{2(\sigma_{bz}^2+\sigma_{lz}^2)}}$$

Maximum total neutralization is achieved when $d = s = 0$.

$$\eta_0 = \frac{\lambda W_l}{2\pi hc}\frac{(1-\beta\cos\theta)}{\beta\sin\theta}\frac{\Sigma_0}{\sigma_{eff}\sqrt{\sigma_{bz}^2+\sigma_{lz}^2}}.$$

Short pulse laser.
Let's consider case when laser pulse is much shorter than ion bunch duration. This situation is typical for bunch length measurement using mode locked laser. Then $\sigma_{ls} << \sigma_{bs}$ and expression for effective longitudinal size becomes:

$$\sigma_{eff} = \sqrt{\frac{\sigma_{bs}^2\sin^2\theta + \sigma_{by}^2(\beta-\cos\theta)^2 + \sigma_{ly}^2(1-\beta\cos\theta)^2}{\beta^2\sin^2\theta}}.$$

In order to reach maximum resolution in longitudinal direction one should minimize contribution from transverse ion beam size. If transverse laser beam size is small compared to transverse ion beam size $\sigma_{ly} << \sigma_{by}$ than minimum is reached at

$$\cos\theta = \beta \qquad (1)$$

and minimum value is $\sigma_{eff} = \frac{\sigma_{bs}}{\beta} = \frac{\tau_b \beta c}{\beta} = \tau_b c$ equal to ion bunch duration. Total neutralization degree in this case is:

$$\eta_0 = \frac{\lambda W_l}{2\pi hc^2}\frac{\sqrt{1-\beta^2}}{\beta}\frac{\Sigma_0}{\tau_b\sigma_{bz}} \quad \text{or}$$

substituting $h = 6.623\text{e-}34$, $c = 2.998\text{e}8$ we obtain the following expression:

$$\eta_0 \approx 2.7\cdot\lambda[\mu m]W_l[mJ]\frac{\sqrt{1-\beta^2}}{\beta}\frac{\Sigma_0[10^{-17}cm^2]}{\tau_b[ps]\sigma_{bz}[mm]},$$

that gives $\eta_0 = 3\cdot 10^{-4}$ for typical mode locked laser parameters of $\lambda = 1.064\mu m$, $W_l = .1\mu J$, and MEBT bunch parameters $\beta = .073$, $\tau_b = 20ps$, $\sigma_{bz} = 2mm$, $\Sigma_0 = 3\cdot 10^{-17}cm^2$. With ion energy increase the angle between the laser beam and ion beam goes down in order to satisfy condition (1). At some point photon energy in the beam rest frame becomes smaller than reaction threshold and cross-section goes to zero for chosen laser wavelength. It might make impractical longitudinal measurements at high energy. Then $\theta = 90^o$ is reasonable choice. Effective longitudinal size in this case is:

$$\sigma_{eff} = \sqrt{\frac{\sigma_{bs}^2 + \sigma_{by}^2 \beta^2}{\beta^2}},$$

total neutralization degree is

$$\eta_0 = \frac{\lambda W_l}{2\pi hc} \frac{\Sigma_0}{\sigma_{bz}\sqrt{\sigma_{bs}^2 + \sigma_{by}^2 \beta^2}} = \frac{\lambda W_l}{2\pi hc} \frac{\Sigma_0}{\beta \sigma_{bz}\sqrt{\tau_b^2 c^2 + \sigma_{by}^2}}$$

substituting $h = 6.623\text{e-}34,\ c = 2.998\text{e}8$ we obtain the following expression:

$$\eta_0 \approx .8 \cdot \lambda[\mu m] W_l[mJ] \frac{\Sigma_0[10^{-17} cm^2]}{\beta \sigma_{bz}[mm]\sqrt{.09 \cdot \tau_b^2[ps^2] + \sigma_{by}^2[mm^2]}},$$

that gives $\eta_0 = 2.3 \cdot 10^{-5}$ for typical mode locked laser parameters of $\lambda = 1.064 \mu m$, $W_l = .1 \mu J$, and HEBT bunch parameters $\beta = .875$, $\tau_b = 20 ps$, $\sigma_{bz} = 2mm$, $\Sigma_0 = 3 \cdot 10^{-17} cm^2$.

Long pulse laser.
The opposite case is when laser pulse is much longer than ion bunch duration. This situation is typical for bunch profile measurement using Q-switched laser. Then $\sigma_{ls} >> \sigma_{bs}, \sigma_{by}, \sigma_{ly}$ and expression for effective longitudinal size becomes:

$$\sigma_{eff} \approx \sigma_{ls}$$

Total neutralization degree in this case is:

$$\eta_0 = \frac{\lambda W_l}{2\pi hc} \frac{(1 - \beta\cos\theta)}{\beta\sin\theta} \frac{\Sigma_0}{\sigma_{ls}\sqrt{\sigma_{bz}^2 + \sigma_{lz}^2}} \approx \frac{\lambda W_l}{2\pi hc} \frac{(1 - \beta\cos\theta)}{\beta\sin\theta} \frac{\Sigma_0}{\sigma_{ls}\sigma_{bz}}.$$

This expression can be used for neutralization degree calculation only for relatively low laser power levels. If calculated total neutralization degree exceeds about 10% saturation effect becomes important and numerical integration should be used.

Appendix.

To calculate photo-detachment cross-section the following formula based on interpolation of experimental data can be used :

$$\sigma[cm^2] = 7.96\cdot 10^{-17}\cdot e^{\frac{-.514}{\lambda[\mu m]}}(1 - e^{\frac{-3.72}{\lambda[\mu m]}+2.25}),$$

where $\lambda = \dfrac{\lambda_0}{\gamma(1-\beta\cos\theta)}$ is laser wavelength in the ion rest frame, λ_0 is laser wavelength in laboratory frame of reference.

References.

1. R.E.Shafer , TN:LANSCE-1:99-101
2. L.Landau, E.Lifshitz, Theory of fields
3. R.E.Shafer, BIW’98