

X-ray and Neutron Scattering from Crystalline Surfaces and Interfaces

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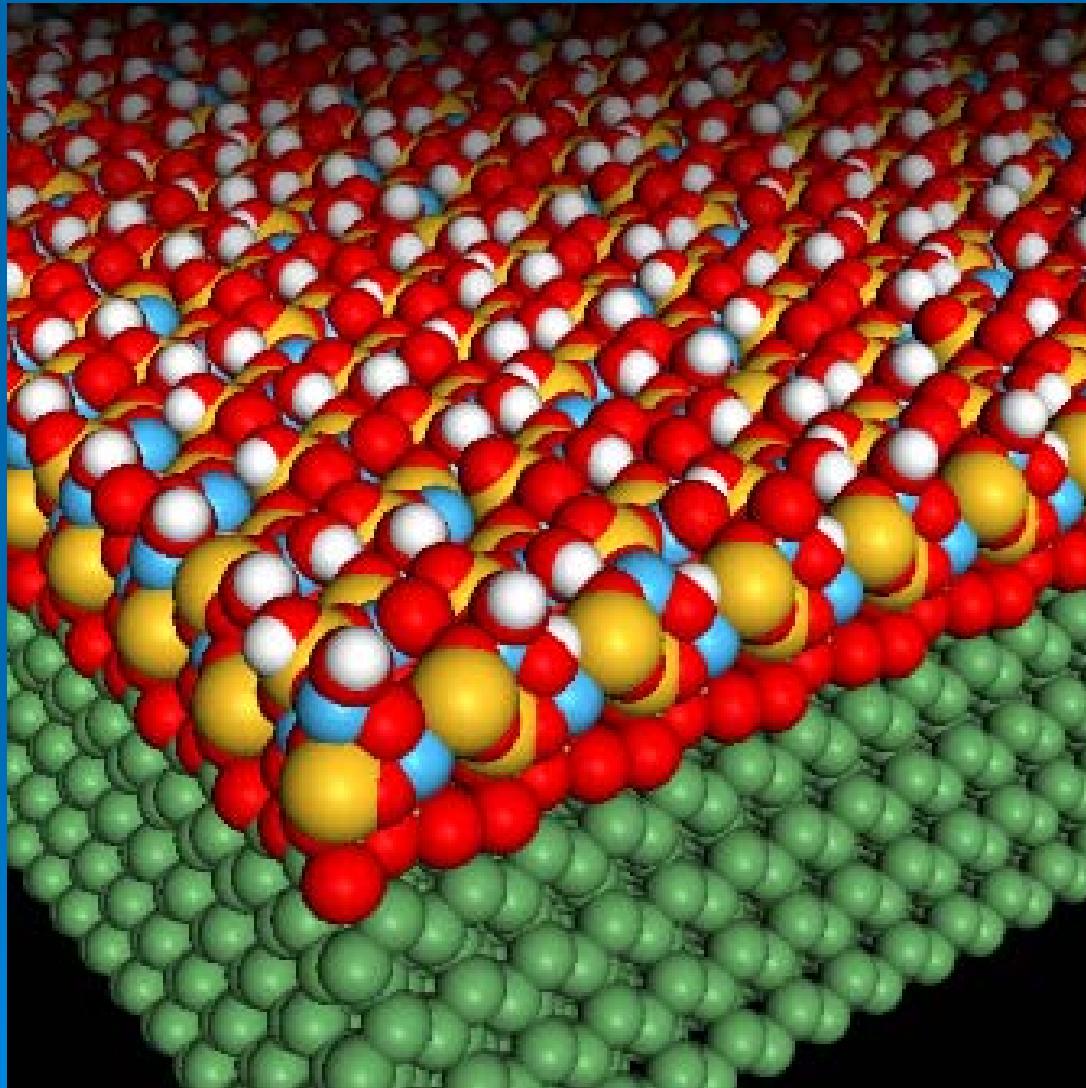
National School for X-ray and Neutron Scattering
Argonne and Oak Ridge National Laboratories

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Many thanks to students and collaborators:
W. Elliott, C. Botez, S.W. Han, M. Gramlich, S. Hayden, Y. Chen,
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M. Tringides; and to NSF and DOE for funding.

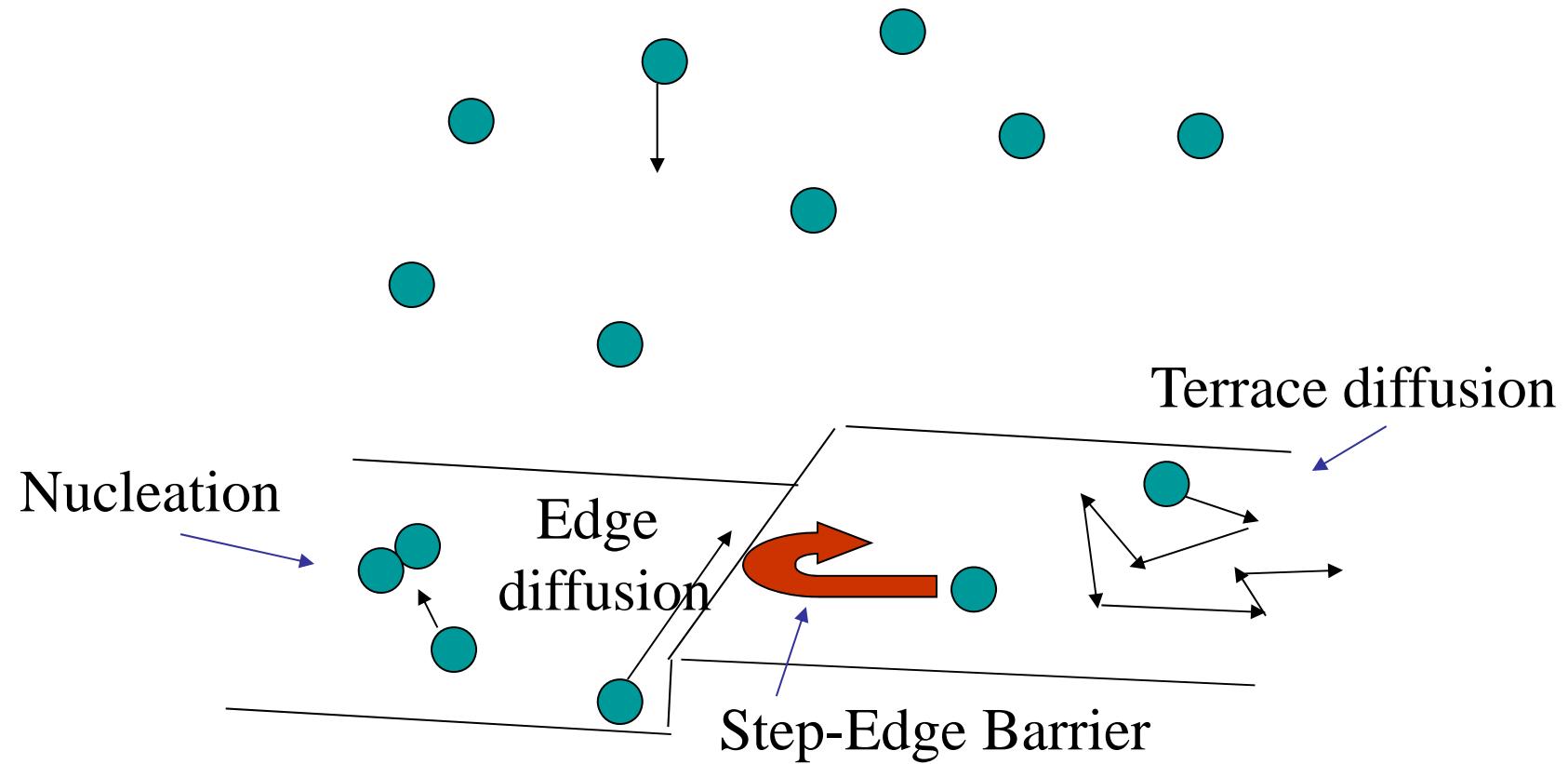
Buried Interface Structure

to understand the growth and function of materials

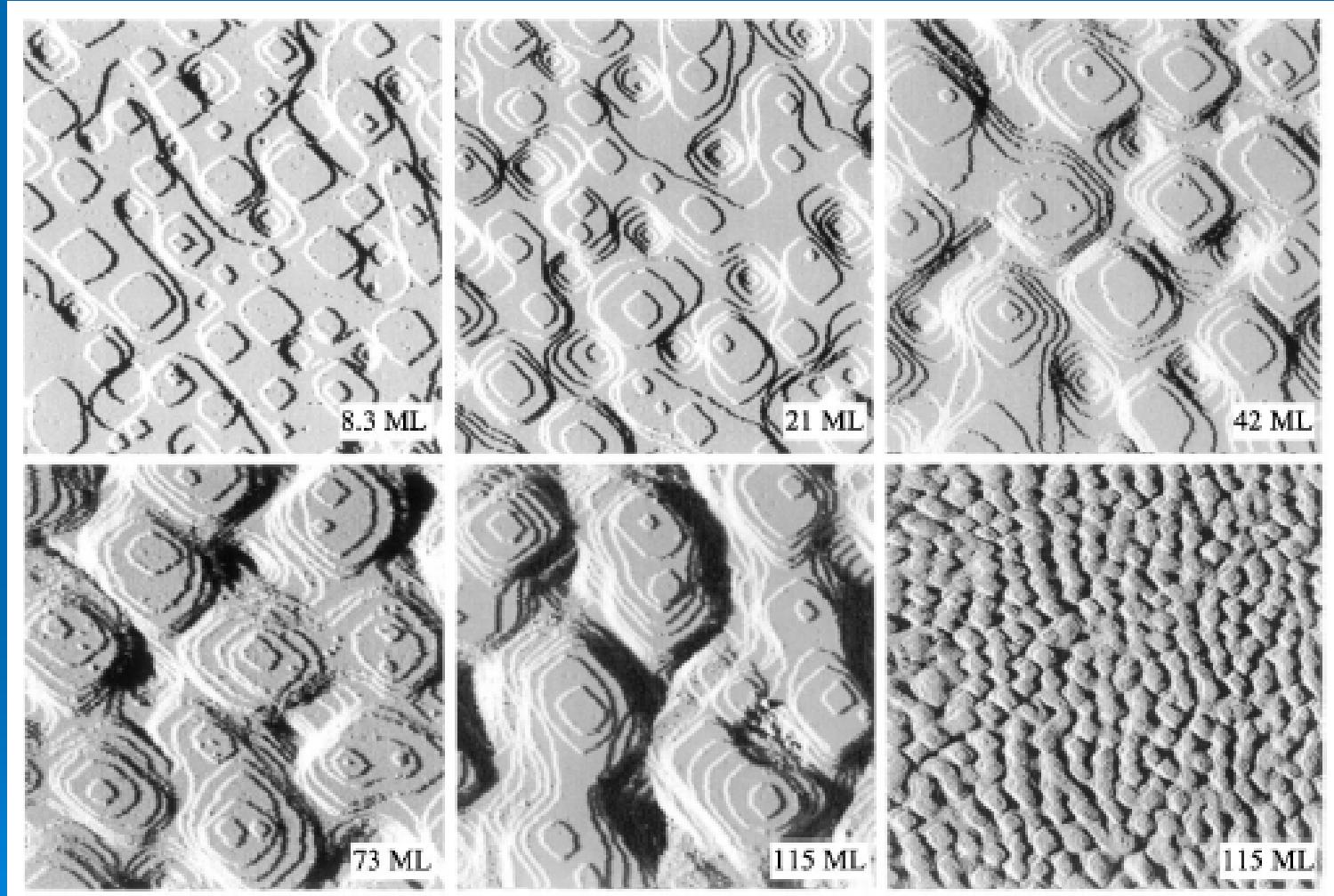


http://www.tyndall.ie/research/electronic-theory-group/thin_film_simulation.html

Crystal growth from a vapor



Morphology → atomic scale mechanisms



Cu/Cu(001)

Zuo & Wendelken

Interplay between two regimes of Length Scales

- Interatomic distances
 - Structure, physics, chemistry → Mechanisms
- “Mesoscale” – Nanoscale
 - Morphology → Mechanisms

Unique Advantages of X-ray Scattering:

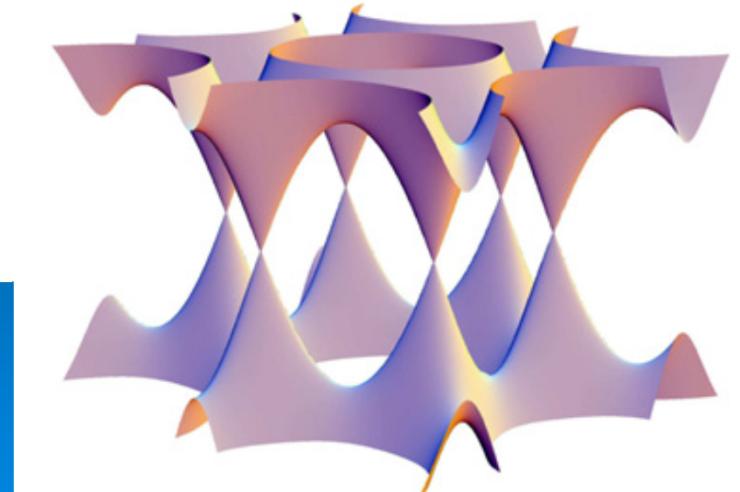
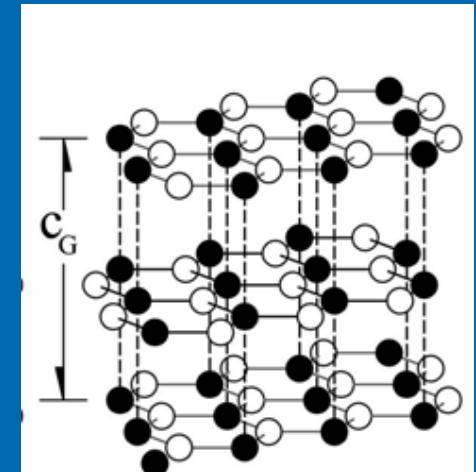
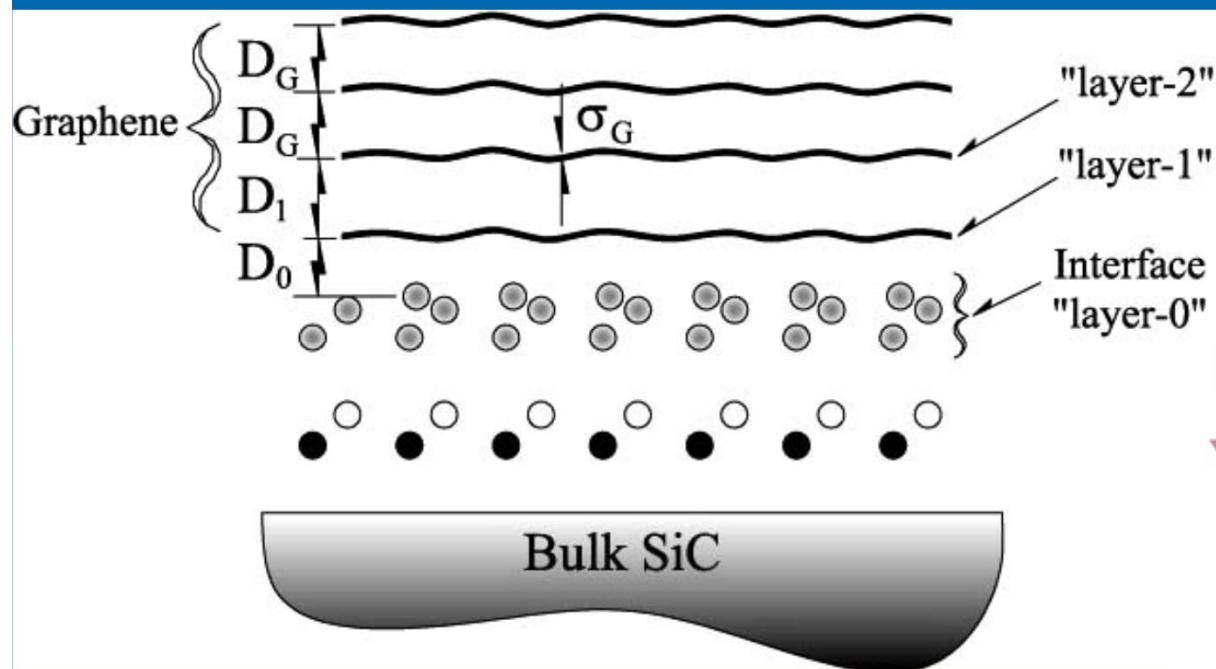
- Atomic-scale structure at a buried interface
- Morphological structure at buried interfaces
- Subsurface phenomena
 - Strains and defects near a surface
- Accurate statistics of distributions
 - (eg. Island size distributions)

Neutrons: low intensity- limited to reflectivity

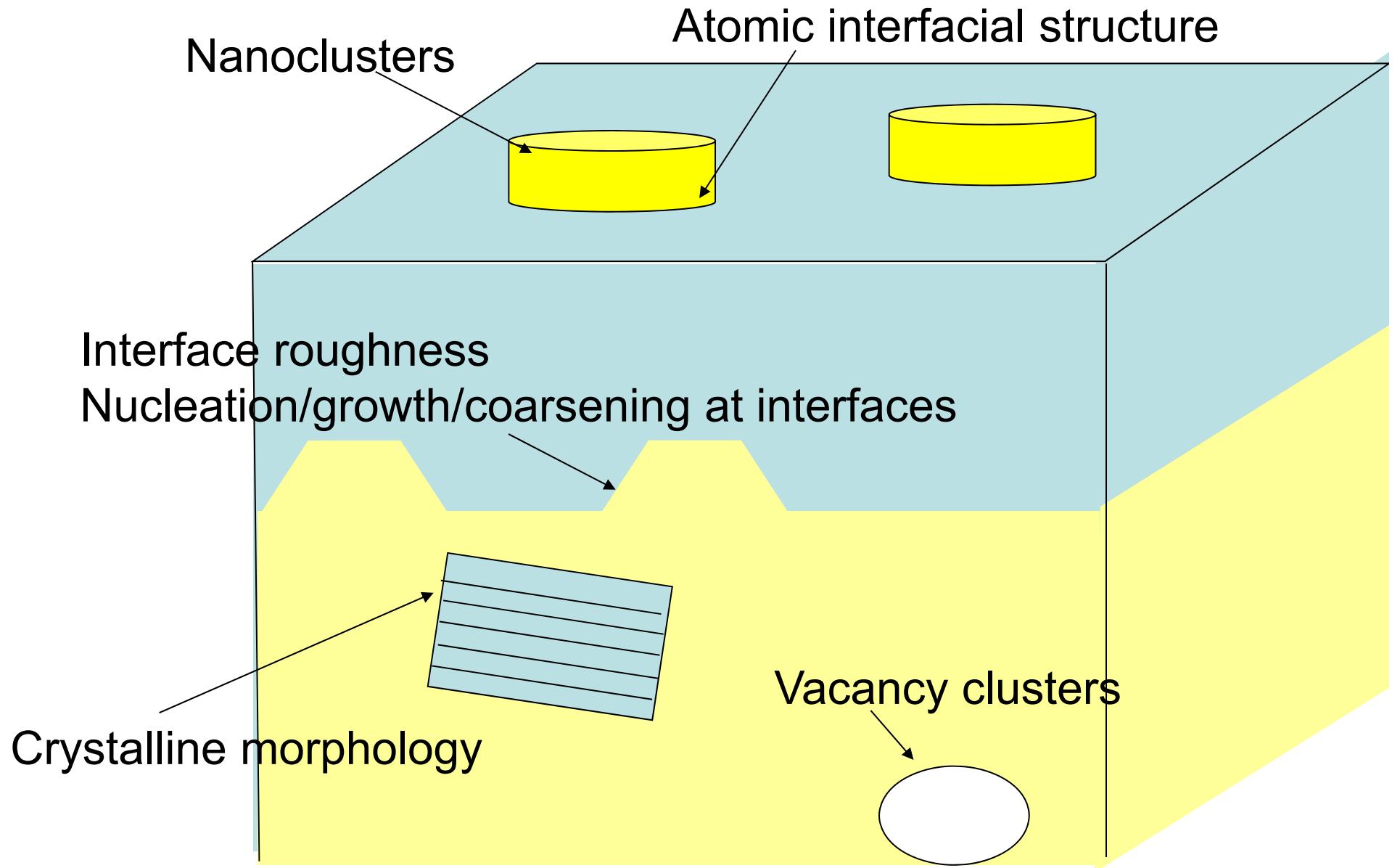
- Soft Matter and Bio materials; H_2O & D_2O
- Magnetic materials

Example: Rotation of graphene planes affect electronic properties

Graphene made from SiC



Morphology → atomic scale mechanisms



Objective

- An introduction to surface scattering techniques
Build a conceptual framework
- Reciprocal Space is a large place: where do we look?

Scattering of X-rays and Neutrons: Helmholtz Equation

$$k = \frac{2\pi}{\lambda}$$

X-rays

$$\nabla^2 \vec{E} + k^2 n^2(\vec{r}) \vec{E} = 0$$

Neutrons

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} [E - V(\vec{r})] \Psi = 0$$

$n(r) = \underline{\text{inhomogeneous}}$ refractive index

Refractive Index for neutron:

$$n(\vec{r}) = \sqrt{1 - \frac{2m}{\hbar^2 k^2} V(\vec{r})} = \sqrt{1 - \lambda^2 \rho_b(\vec{r})/\pi}$$

Scattering length density:

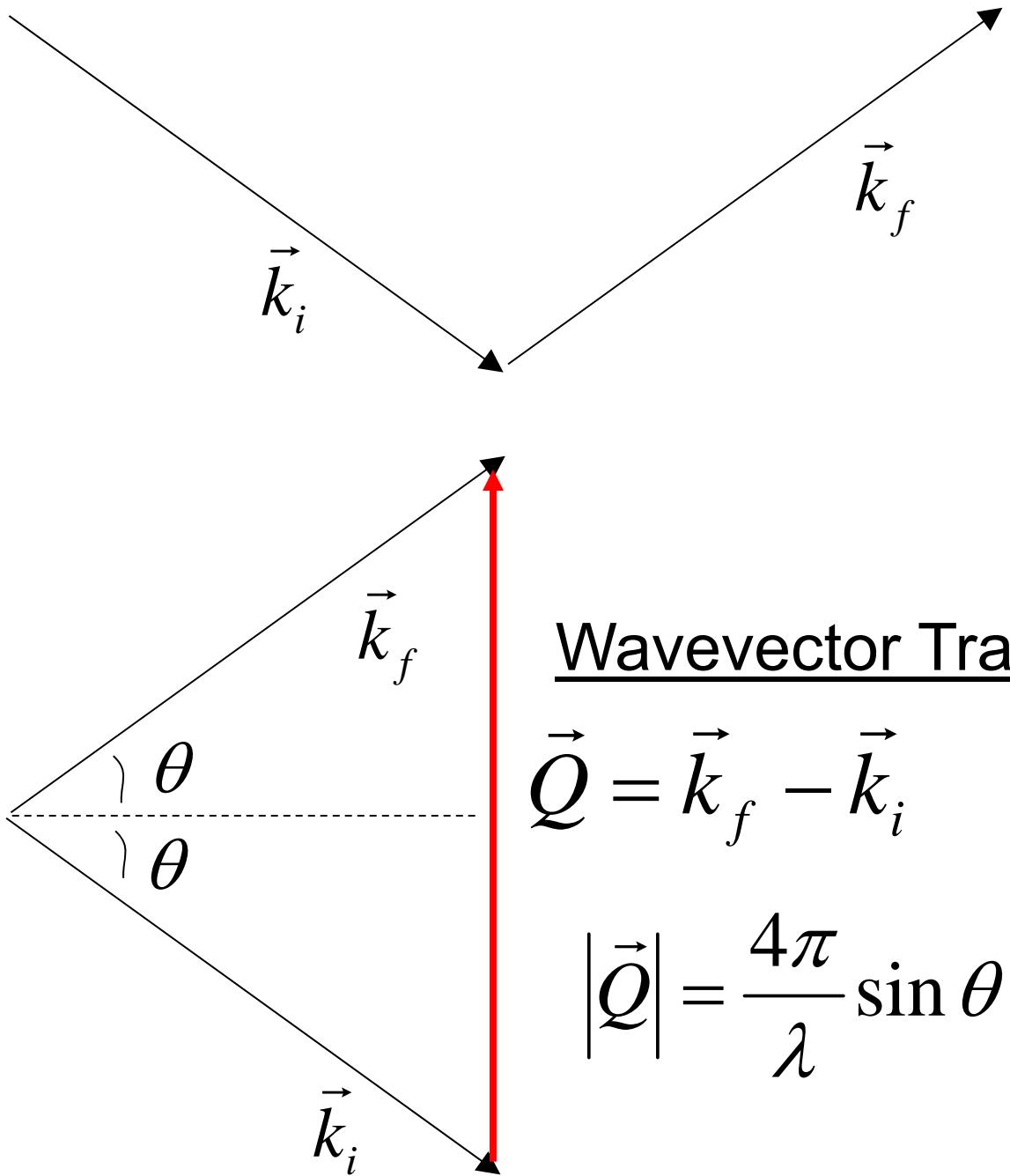
$$\rho_b(\vec{r}) \xrightarrow{\text{monoatomic}} \rho_N(\vec{r}) b$$

number density

scattering length:

$$b = \begin{cases} r_e f(Q) & x-rays \\ \text{tabulated} - \text{neutrons} & \end{cases}$$

One language for both x-rays and neutrons

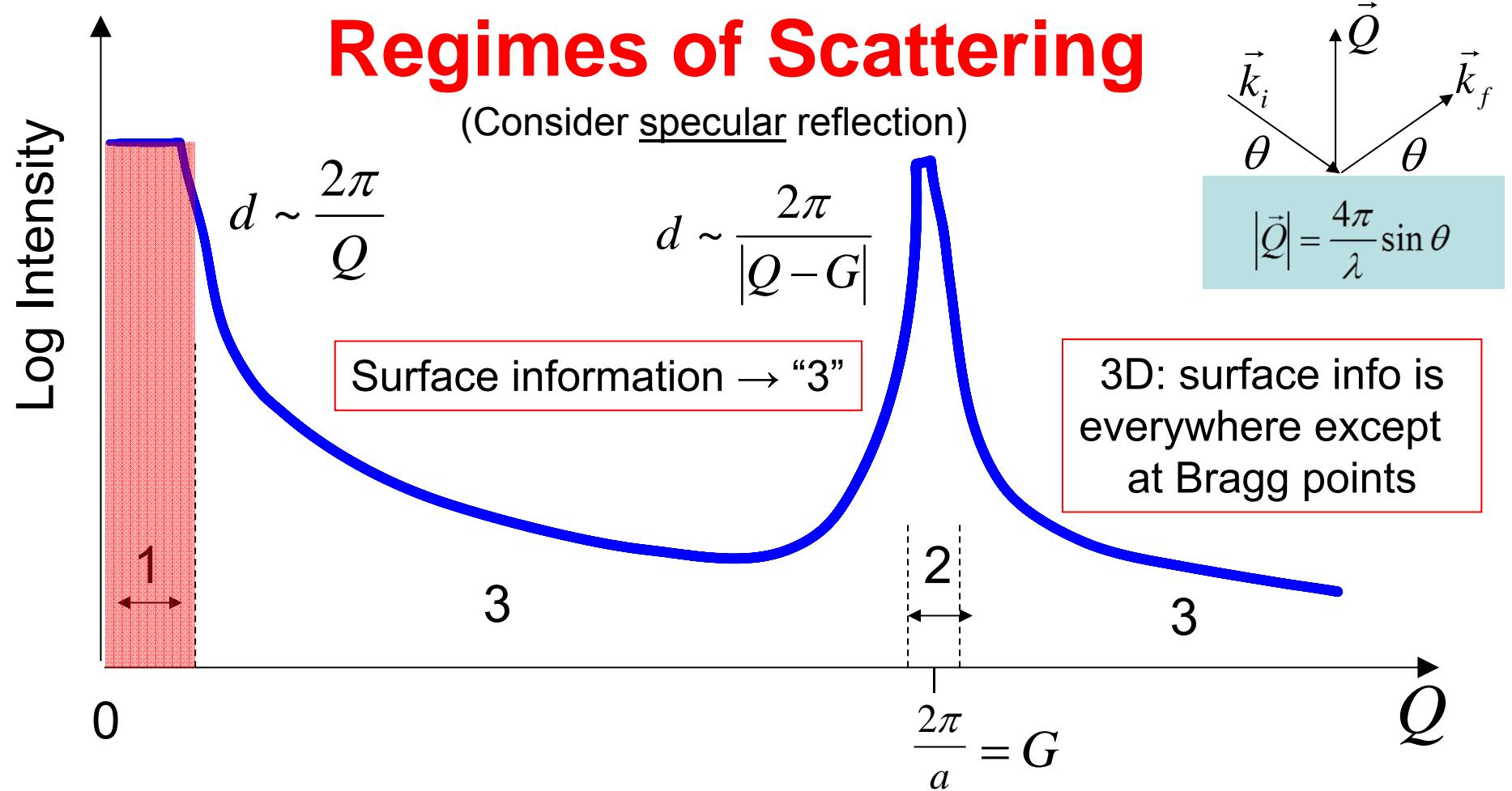


Wavevector Transfer:

$$\vec{Q} = \vec{k}_f - \vec{k}_i$$

$$|\vec{Q}| = \frac{4\pi}{\lambda} \sin \theta \sim \frac{2\pi}{d}$$

Probing Length Scale



1. Grazing angle reflectivity: **strong scattering** $d \gg$ interatomic distances
Exact solution required. Neglect atomic positions: homogeneous medium
2. Bragg region: **strong scattering**; $d \sim$ interatomic distances = a
Exact solution required. Atomic positions needed. Similar to e^- band theory.
3. Everywhere else: **weak scattering**
Born approximation → simplification. Atomic positions required.

Grazing Angles: Refraction and Total Reflection

$d \gg a$: consider homogenous medium

Use average refractive index:

$$n = \sqrt{1 - \lambda^2 \rho_b / \pi} \equiv 1 - \delta \quad \delta = \frac{\lambda^2 \rho_b}{2\pi} \ll 1 \quad (\sim 10^{-5})$$

With absorption: $n = 1 - \delta - i\beta$

Snell's Law:

$$\sin^2 \theta' = \sin^2 \theta - 2\delta$$

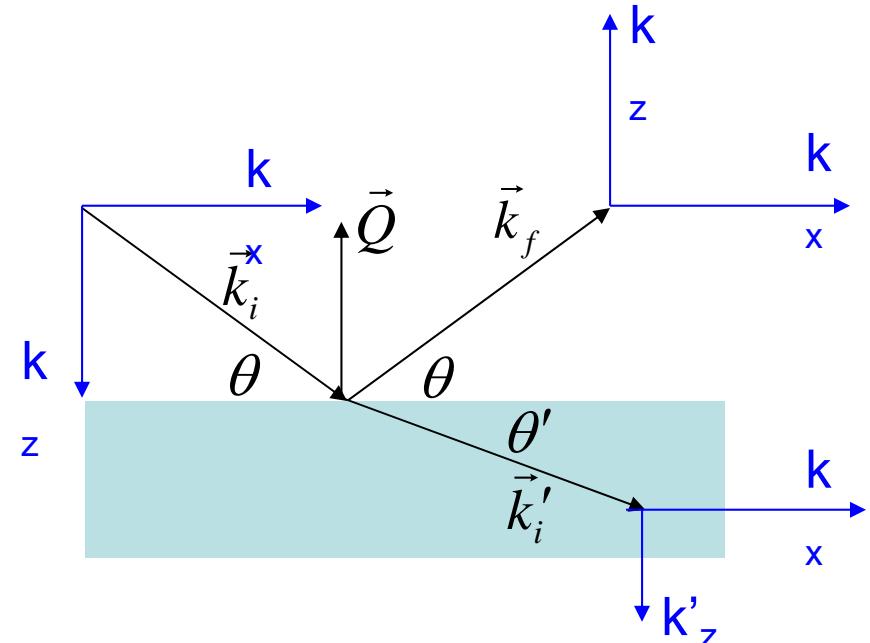
Critical Angle for
Total Reflection:

$$\theta_c = \sqrt{2\delta}$$

Wavevector transfer:

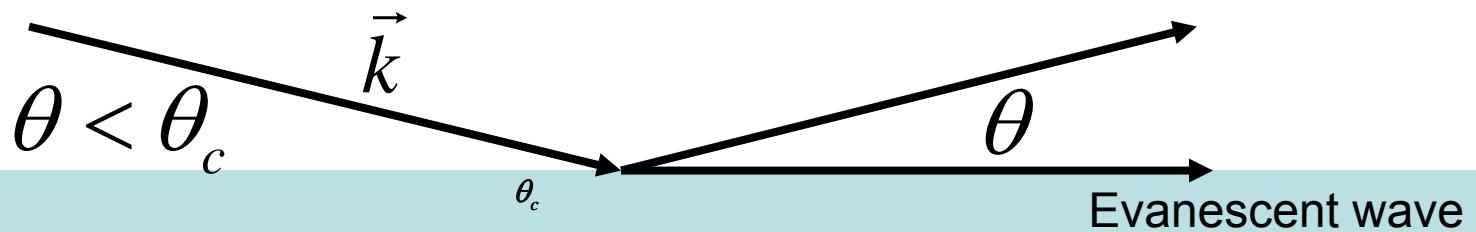
$$Q'^2 = Q^2 - Q_c^2$$

$$Q_c^2 = 16\pi\rho_b$$



Only k_z component is affected by the surface.
 k_x is unchanged.

Total Reflection



$$\sin^2(\theta') = 0 = \sin^2(\theta_c) - 2\delta$$

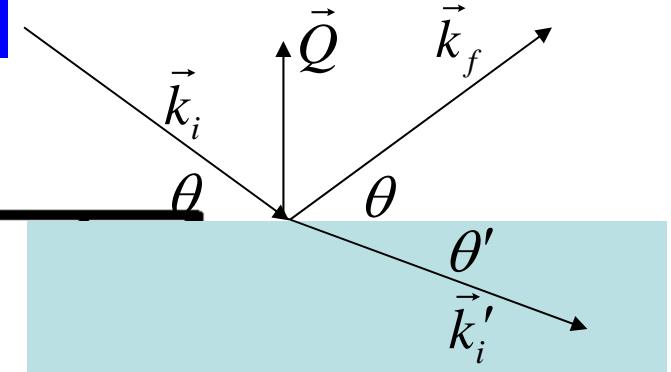
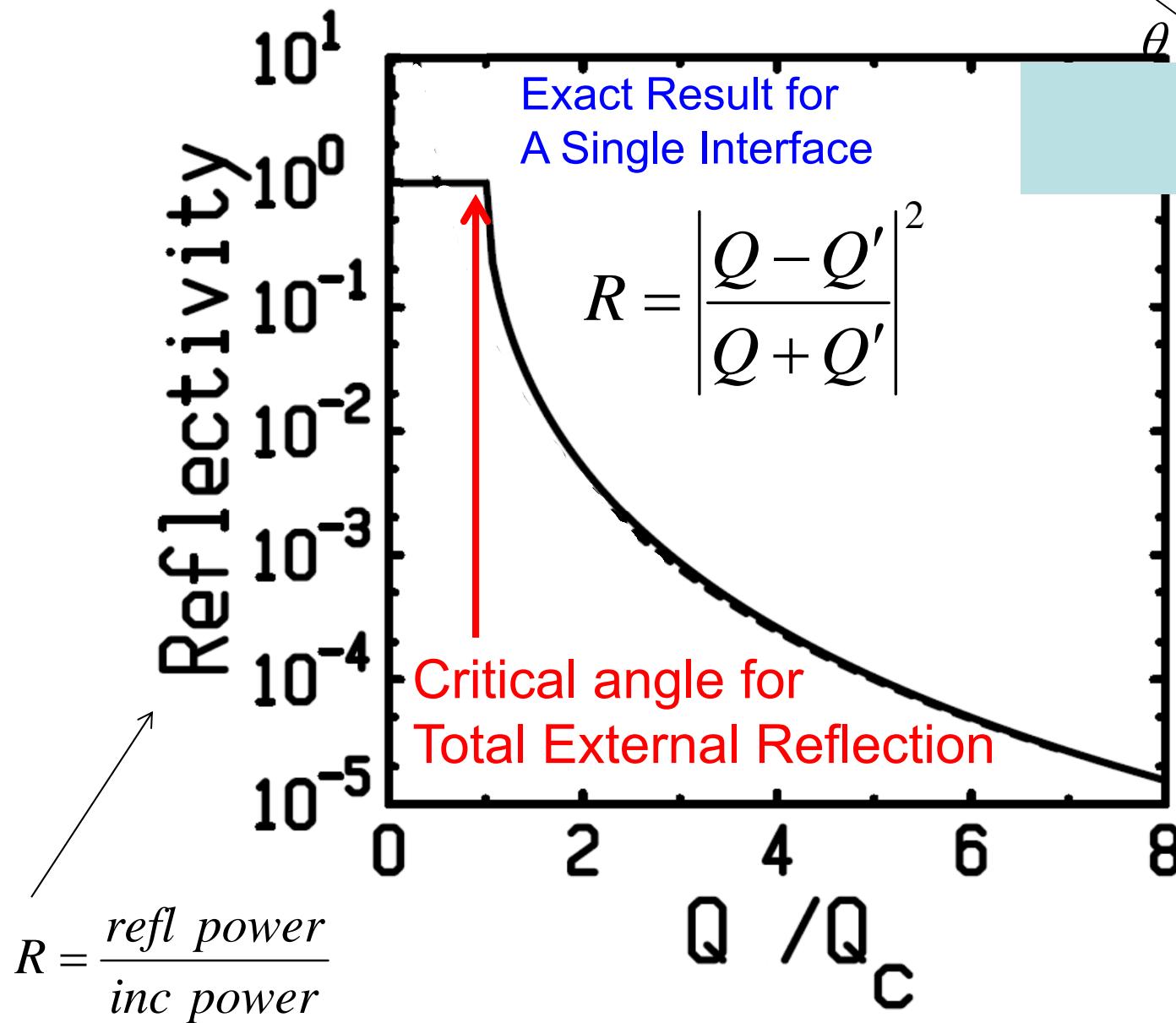
Critical Angle for Total External Reflection:

$$\theta_c = \sqrt{2\delta}$$

$$Q_c = \frac{4\pi}{\lambda} \theta_c$$

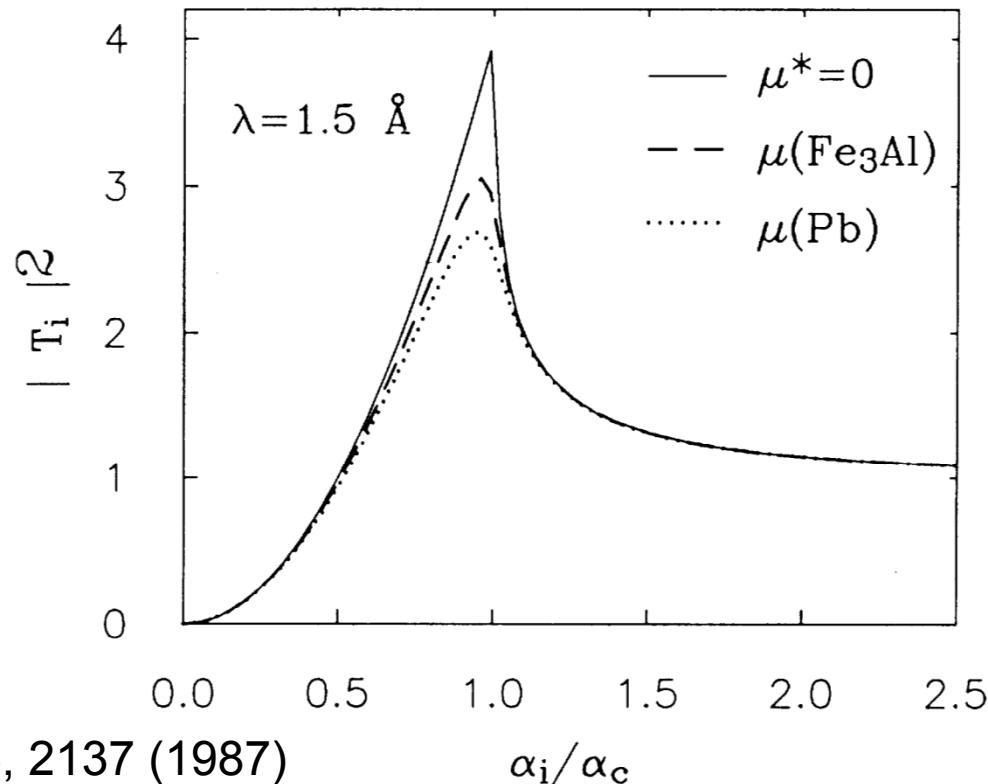
Beam does not transmit below θ_c

Fresnel Reflectivity for a Single Interface



Transmission Amplitude

$$T_i = \frac{2 \sin \alpha_i}{\sin \alpha_i + (\sin^2 \alpha_i - 2\delta)^{1/2}}$$



H. Dosch, PRB 35, 2137 (1987)

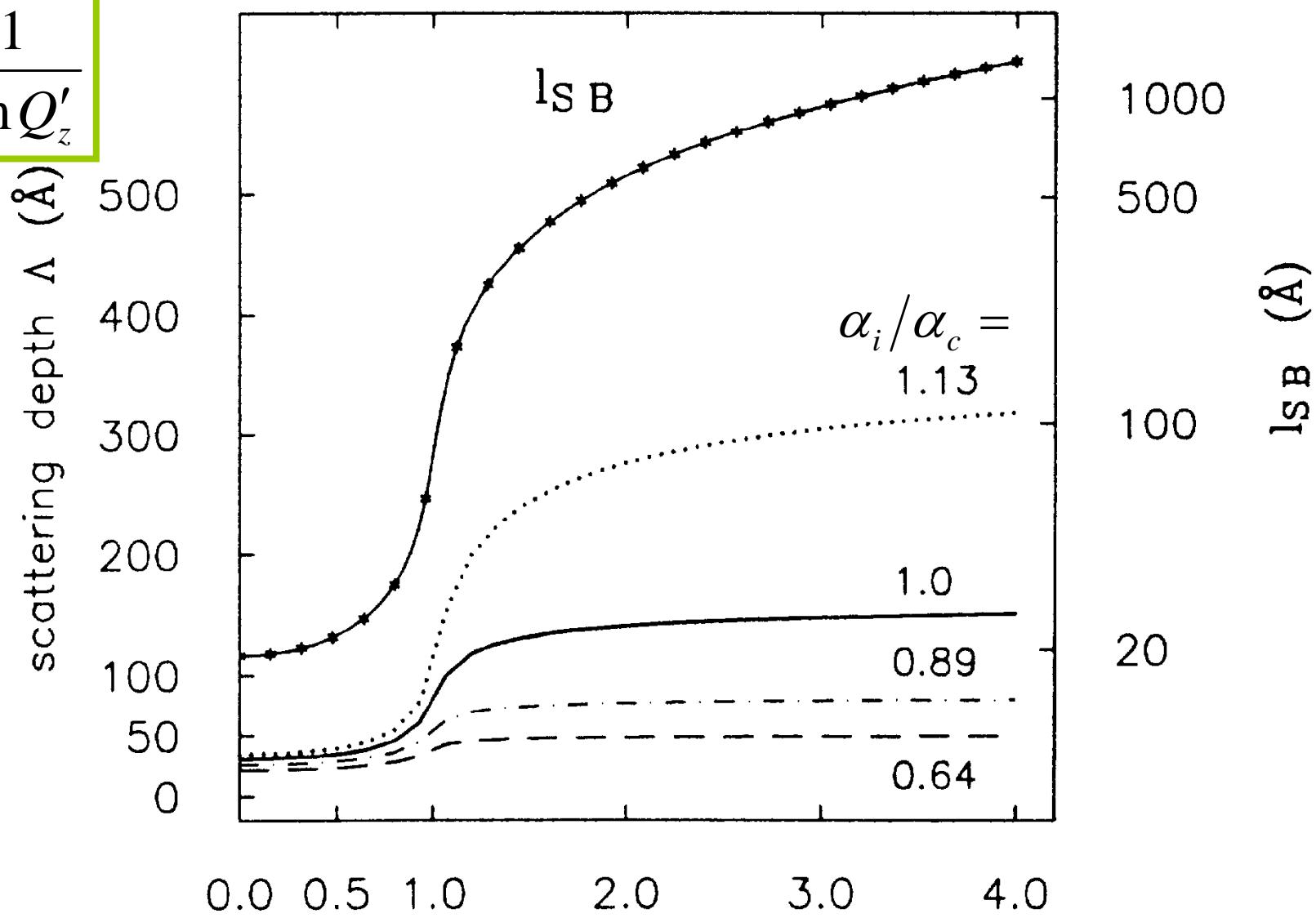
FIG. 1. Fresnel transmissivity $|T_i|^2$ as a function of α_i/α_c for a transparent medium and the real systems Fe_3Al and Pb .

$$\Lambda = \lambda / [2\pi(l_i + l_f)]$$

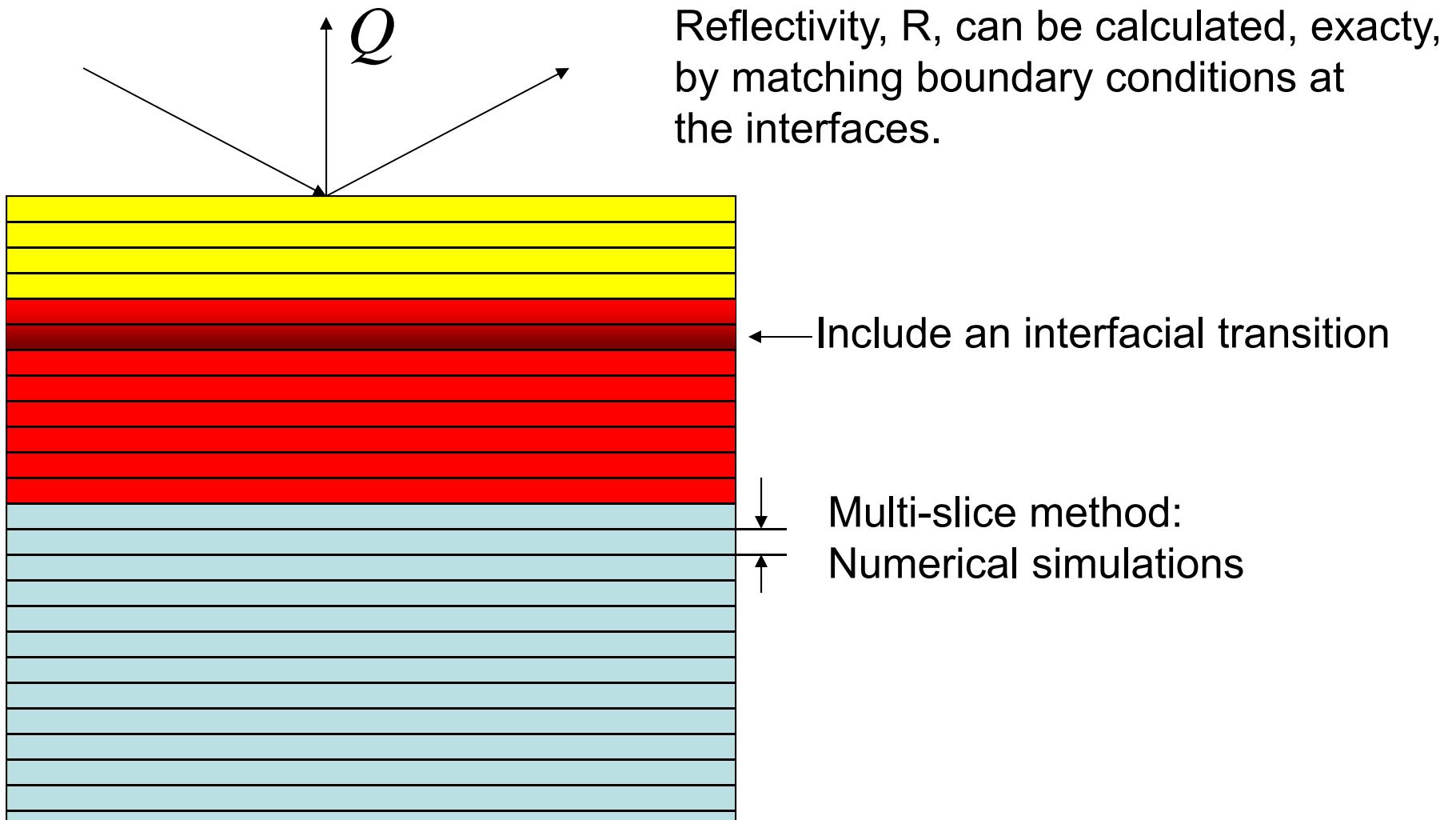
Penetration Length

$$l_{i,f} = \frac{1}{2}\sqrt{2}\{(2\delta - \sin^2\alpha_{i,f}) + [(\sin^2\alpha_{i,f} - 2\delta)^2 + (2\beta)^2]^{1/2}\}^{1/2}$$

$$\boxed{\Lambda = \frac{1}{\text{Im } Q'_z}}$$



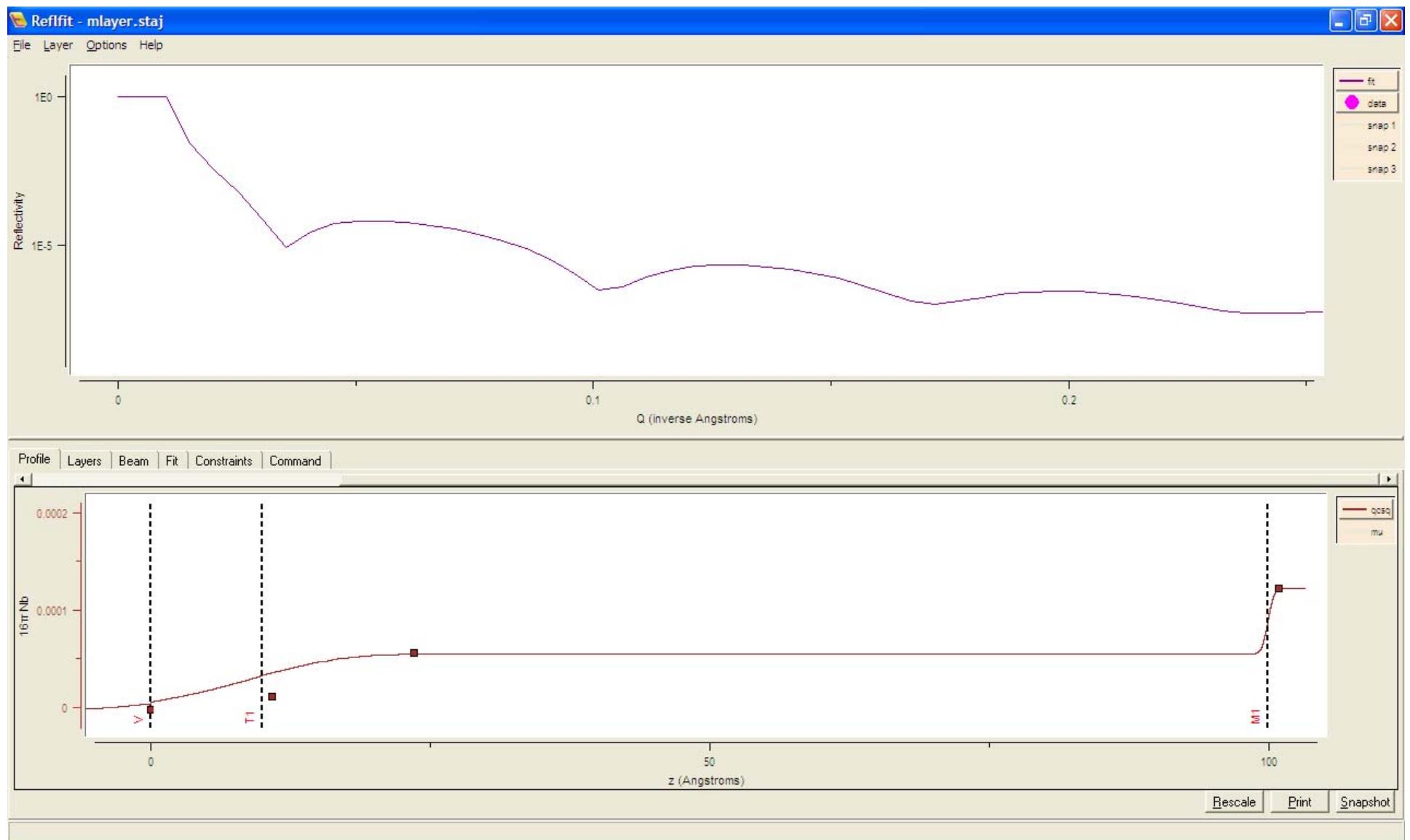
Calculation of reflectivity



L.G. Parratt, Phys Rev 95, 359 (1954)

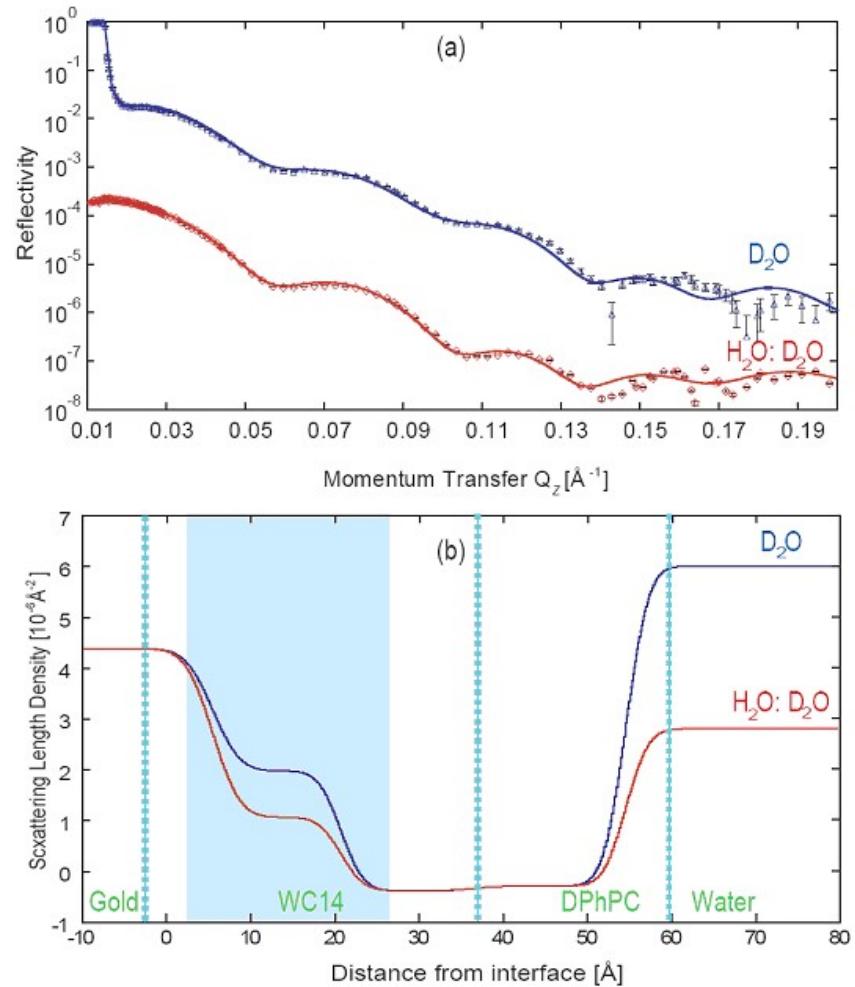
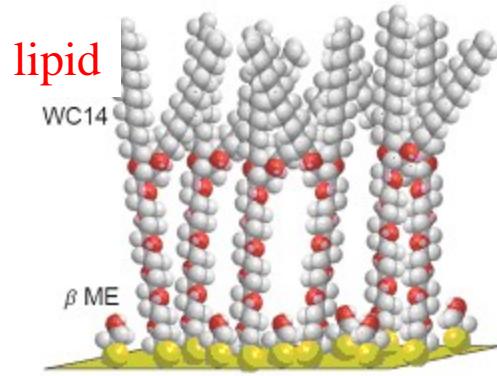
M-layer

<http://www.ncnr.nist.gov/reflpak/>



Soft Matter: Neutron Reflectivity

- Thin film interfaces
- Light elements
- Isotopic substitutions
- Polymers



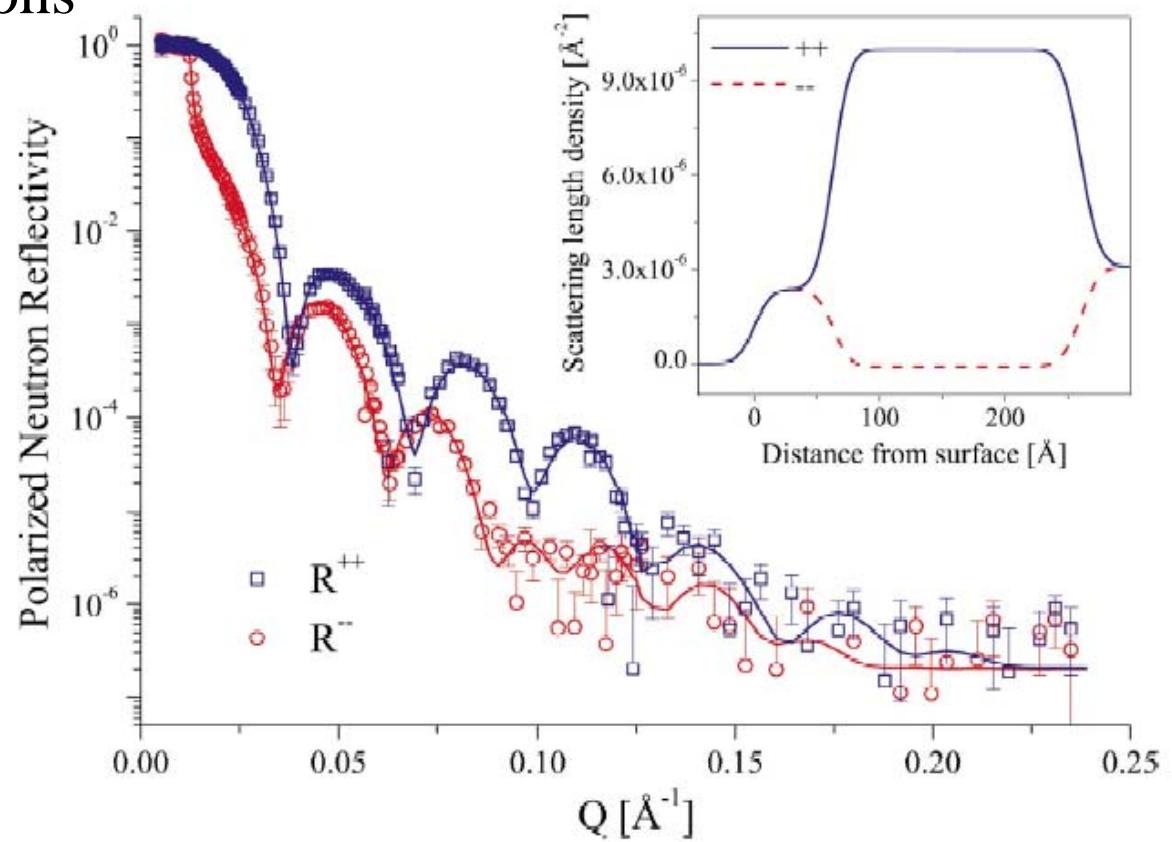
D. J. McGillivray and M. Lösche
D. J. Vanderah and J. J. Kasianowicz
G. Valincius

Magnetic Films: Neutron Reflectivity

- Magnetic thin films

Spin-polarized neutrons

FeCo/GaAs

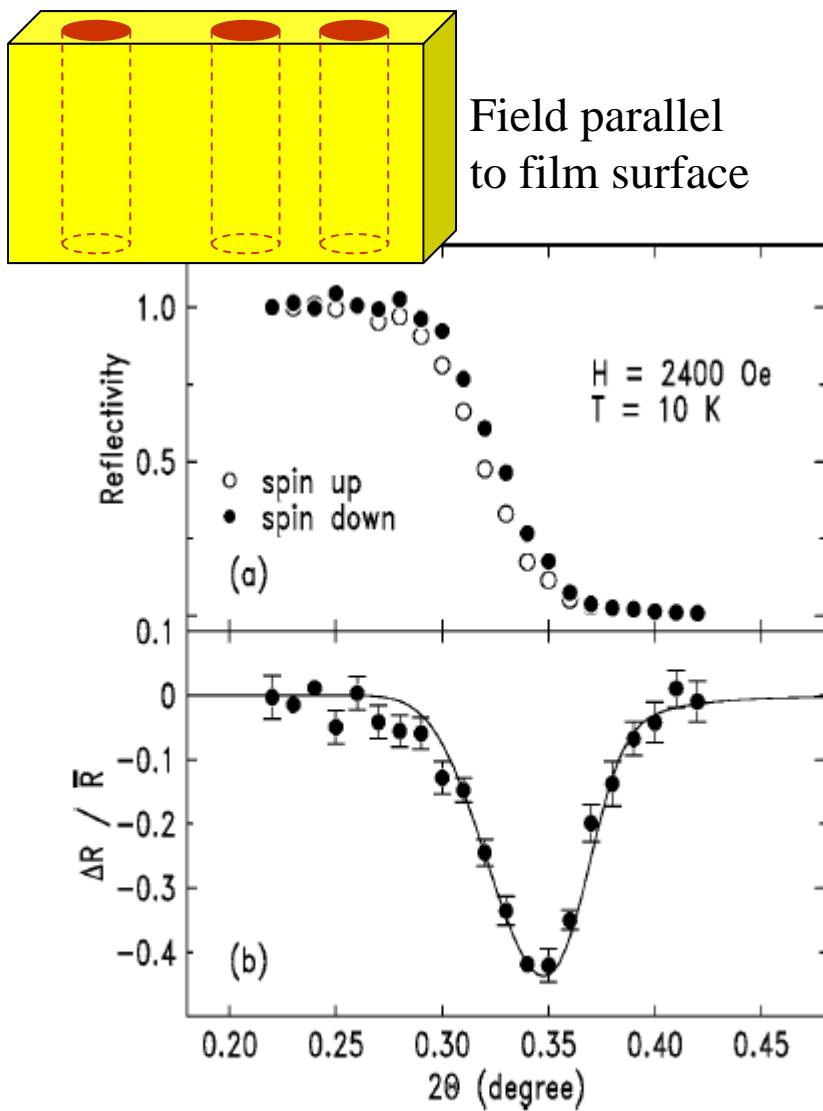


S. Park, M. R. Fitzsimmons, X. Y. Dong,
B. D. Schultz, and C. J. Palmstrøm, Phys Rev B 70 104406 (2004)

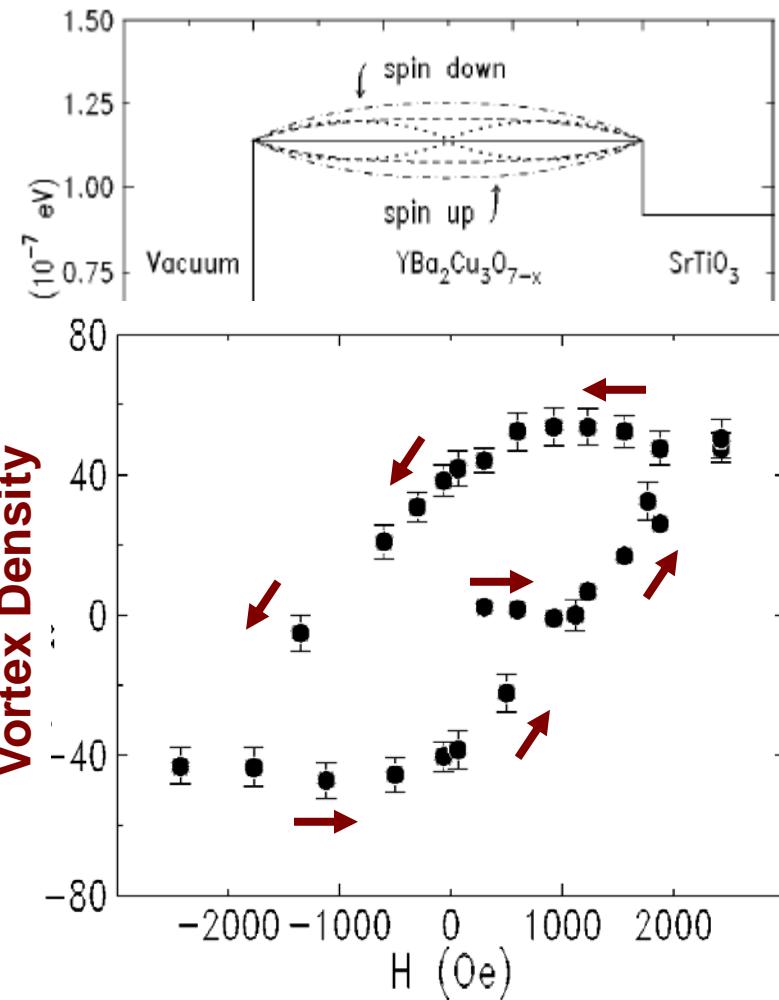
Vortices in Thin-Film Superconductors Studied by Spin-Polarized Neutron Reflectivity (SPNR)

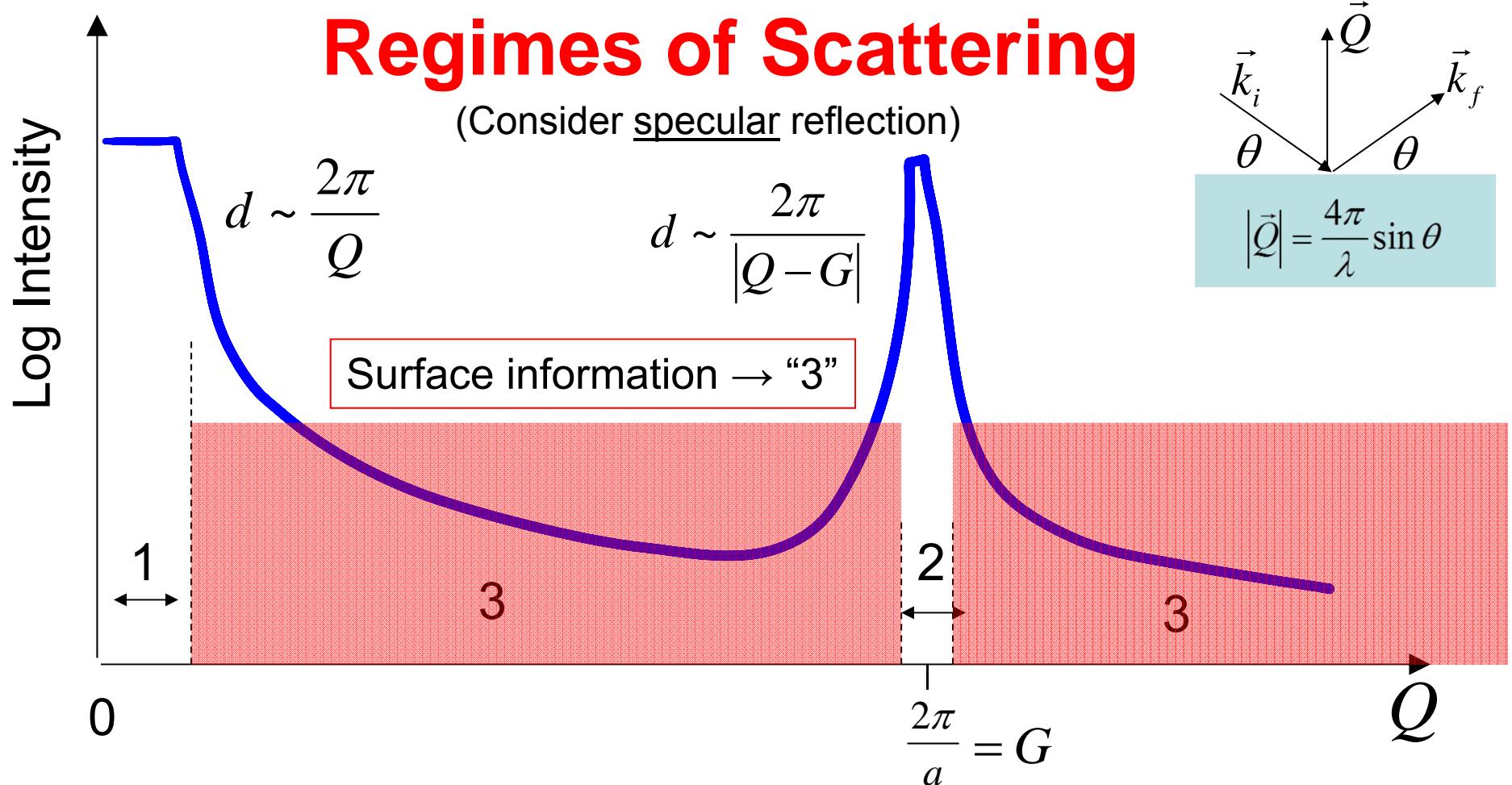
H

S-W. Han, J.F. Ankner, H.Kaiser, P.F.Miceli,
E.Paraoanu, L.H.Greene, PRB **59**, 14692 (1999)



YBCO 600 nm Superconducting Film





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Exact solution required. Neglect atomic positions: homogeneous medium
2. Bragg region: **strong scattering**; $d \sim$ interatomic distances = a
Exact solution required. Atomic positions needed. Similar to e^- band theory.
3. Everywhere else: **weak scattering**
Born approximation → simplification. Atomic positions required.

Differential Scattering Cross Section

Weak Scattering

“Born Approximation” or “Kinematic Approximation”

$$\frac{d\sigma}{d\Omega} = P S(\vec{Q}) = P |A(\vec{Q})|^2$$

P is the polarization factor (x-ray case)

S(Q) is the structure factor

A(Q) is the scattering amplitude

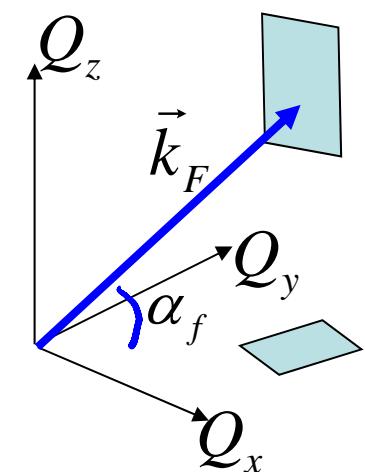
f(Q) is the atomic form factor

ρ_b is the scattering length density

Reflectivity:

$$R = \frac{1}{A_{inc}} \int d\Omega \frac{d\sigma}{d\Omega}$$

$$d\Omega = \frac{d^2 \vec{Q}_p}{k^2 \sin(\alpha_f)}$$



Born Approximation: simple sum over atomic positions

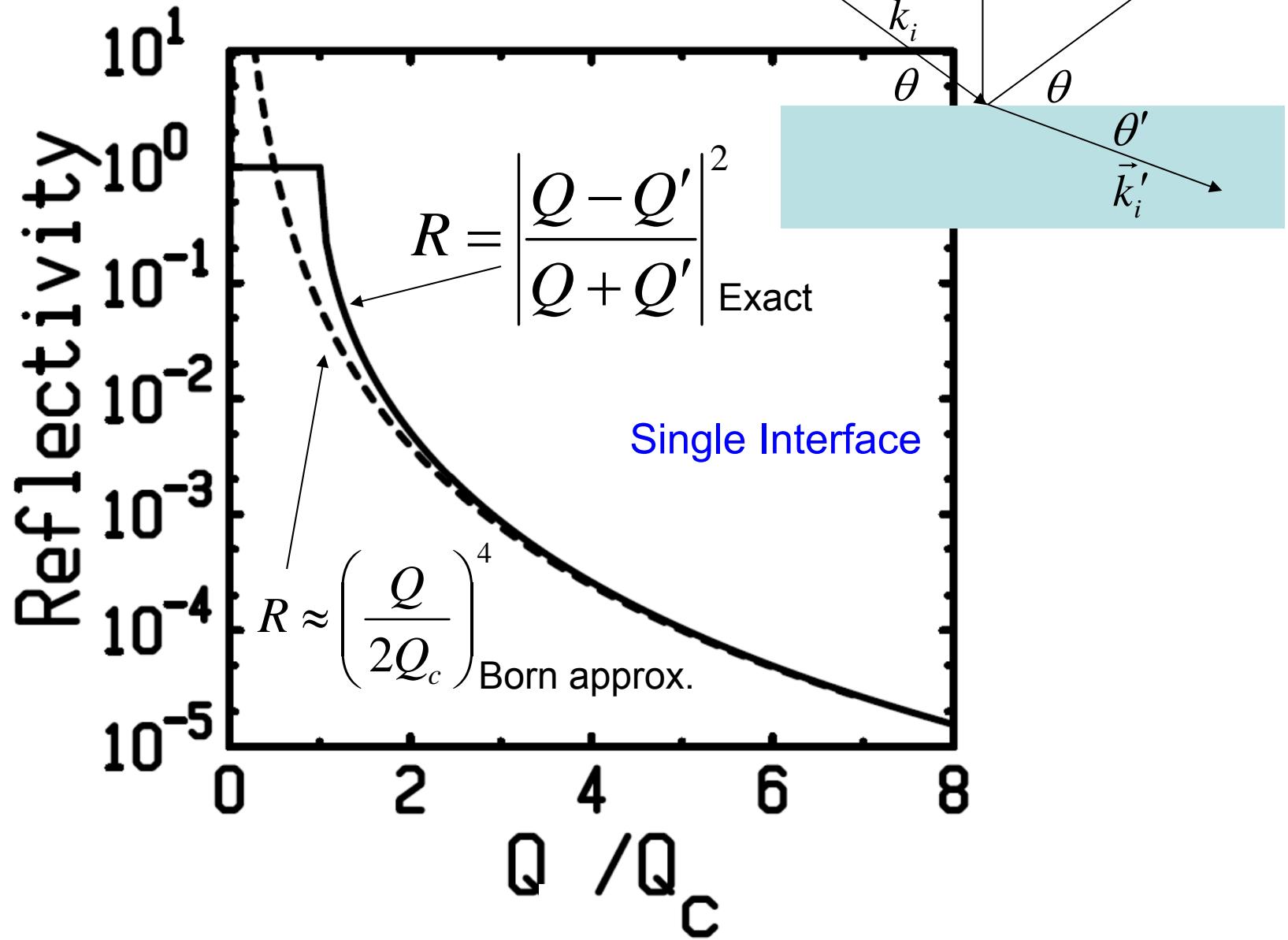
Sum over all atomic positions

$$A(\vec{Q}) = \int d^3\vec{r} \rho_b(\vec{r}) e^{i\vec{Q}\cdot\vec{r}} = \sum_{\vec{r}} b_{\vec{r}} e^{i\vec{Q}\cdot\vec{r}}$$

$b = r_e f(Q)$ for x-rays or tabulated for neutrons

Born Approximation works if the reflectivity is not too large.

Specular Reflection



General Case: non-specular scattering

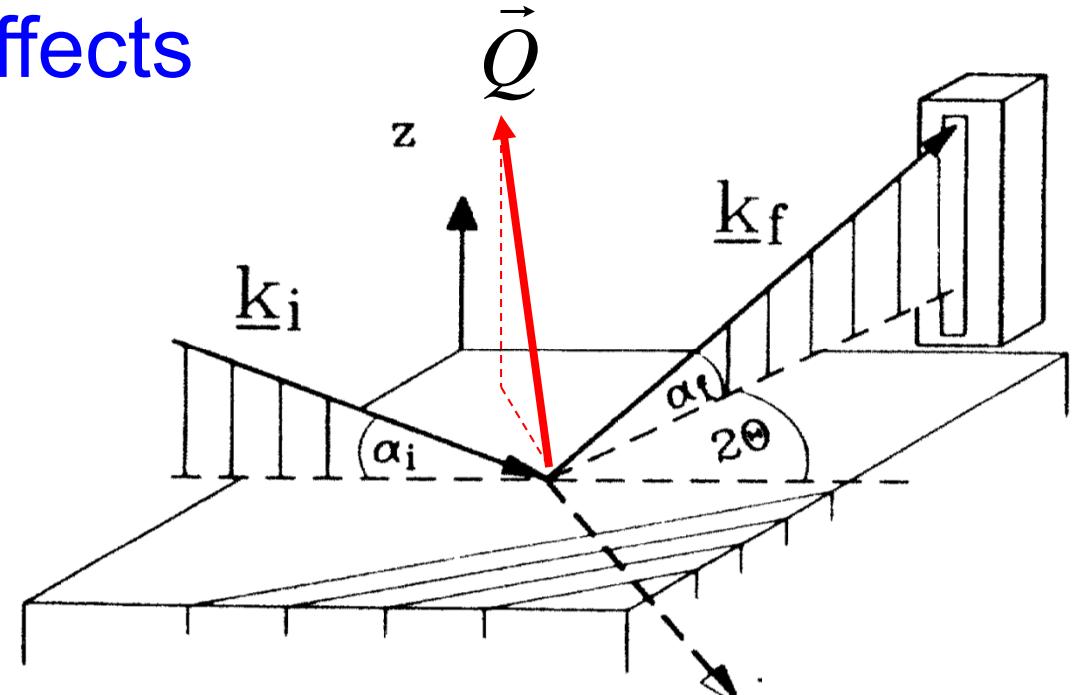
Grazing Incidence Effects

3D Scattering

$$\vec{Q} = \vec{k}_f - \vec{k}_i$$

If \vec{k}_i or \vec{k}_f are near grazing:

- refraction of both beams
internal wavevector transfer: \vec{Q}'
- transmission of both beams: T_i, T_f



Perpendicular to Surface: internal \mathbf{Q}' and external \mathbf{Q} are different

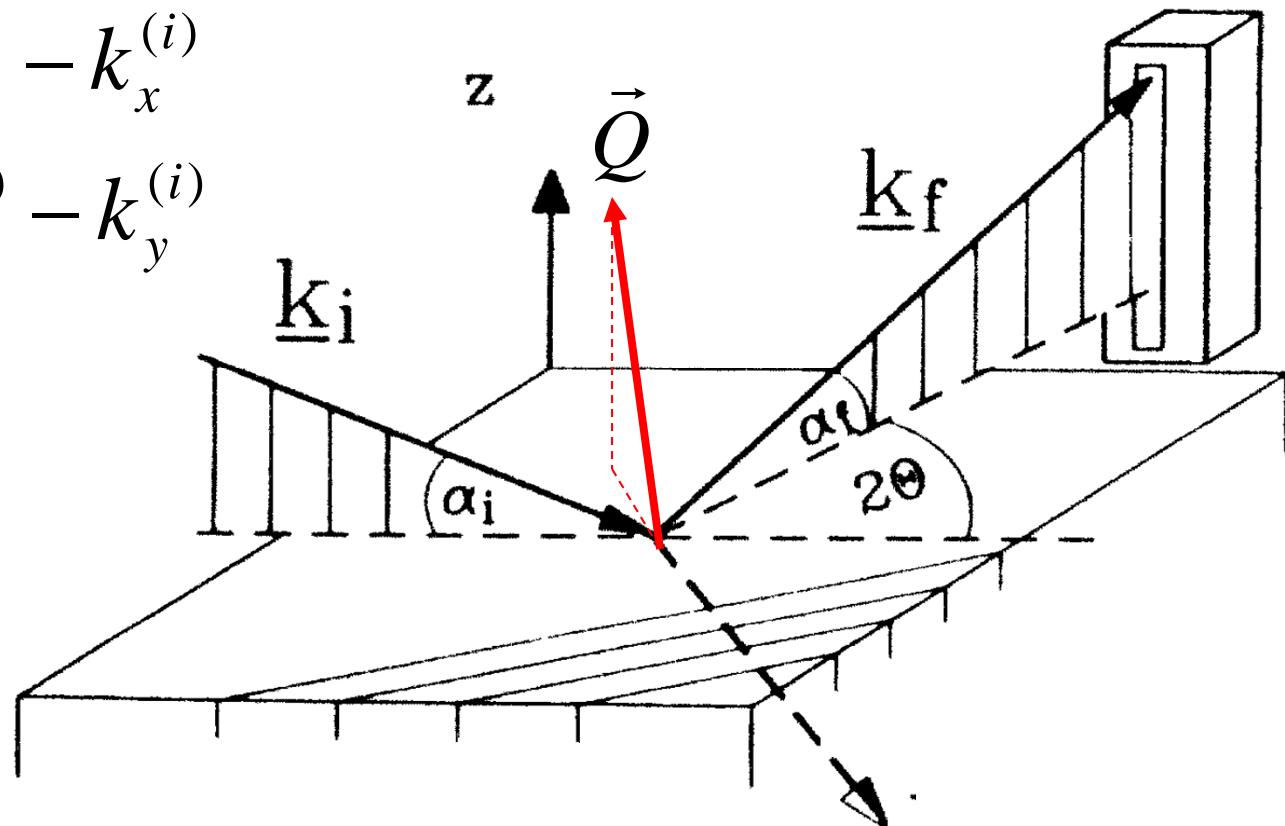
$$Q_z = k_z^{(f)} - k_z^{(i)} = (2\pi/\lambda) [\sin\alpha_f + \sin\alpha_i].$$

$$Q'_z = k_z^{(f)'} - k_z^{(i)'} = (2\pi/\lambda) [(\sin^2\alpha_f - 2\delta - 2i\beta)^{1/2} + (\sin^2\alpha_i - 2\delta - 2i\beta)^{1/2}].$$

Parallel to Surface: internal \mathbf{Q}' and external \mathbf{Q} are same

$$Q'_x = Q_x = k_x^{(f)} - k_x^{(i)}$$

$$Q'_y = Q_y = k_y^{(f)} - k_y^{(i)}$$

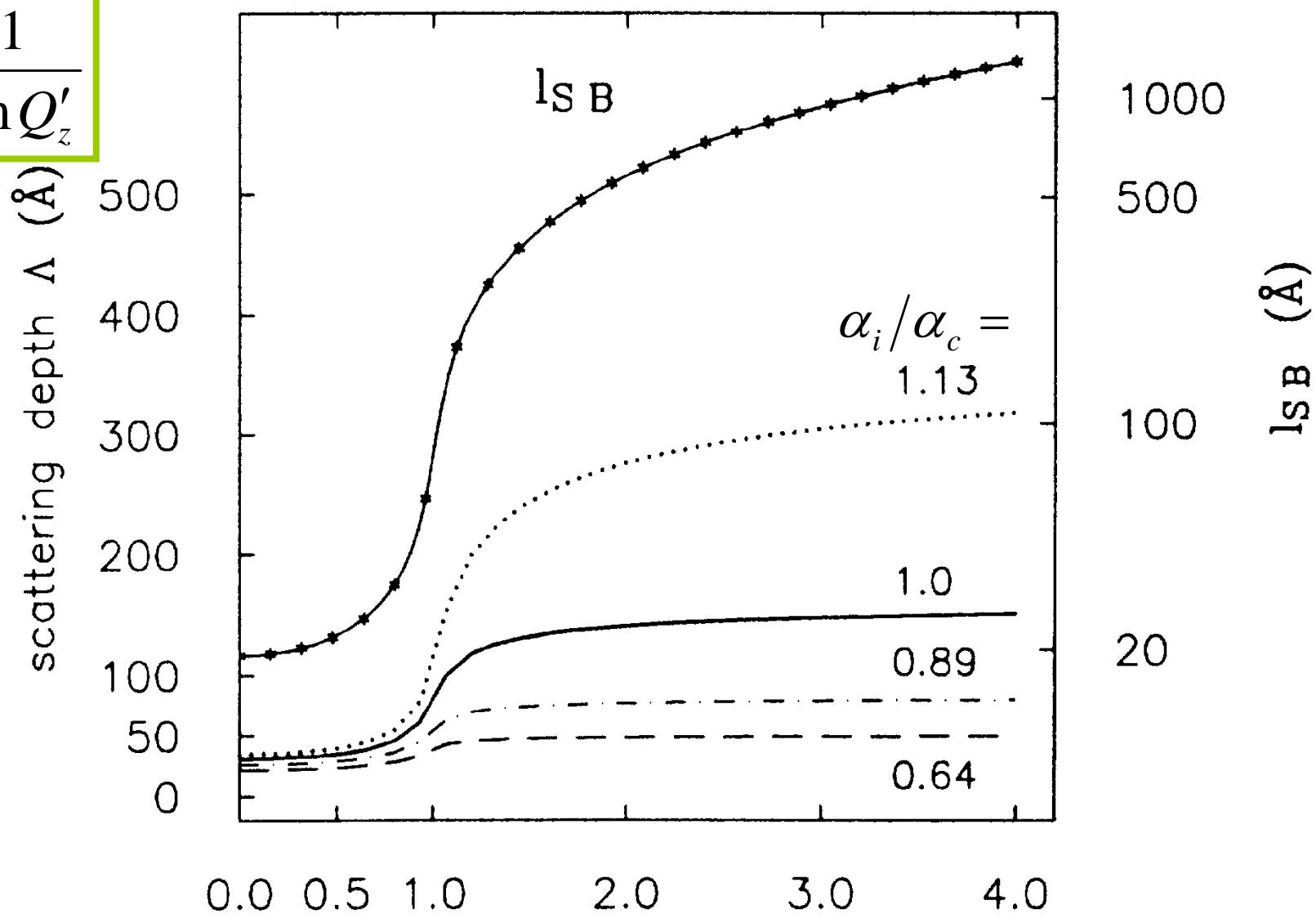


$$\Lambda = \lambda / [2\pi(l_i + l_f)]$$

Penetration Length

$$l_{i,f} = \frac{1}{2}\sqrt{2}\{(2\delta - \sin^2\alpha_{i,f}) + [(\sin^2\alpha_{i,f} - 2\delta)^2 + (2\beta)^2]^{1/2}\}^{1/2}$$

$$\boxed{\Lambda = \frac{1}{\text{Im } Q'_z}}$$



Fe₃Al

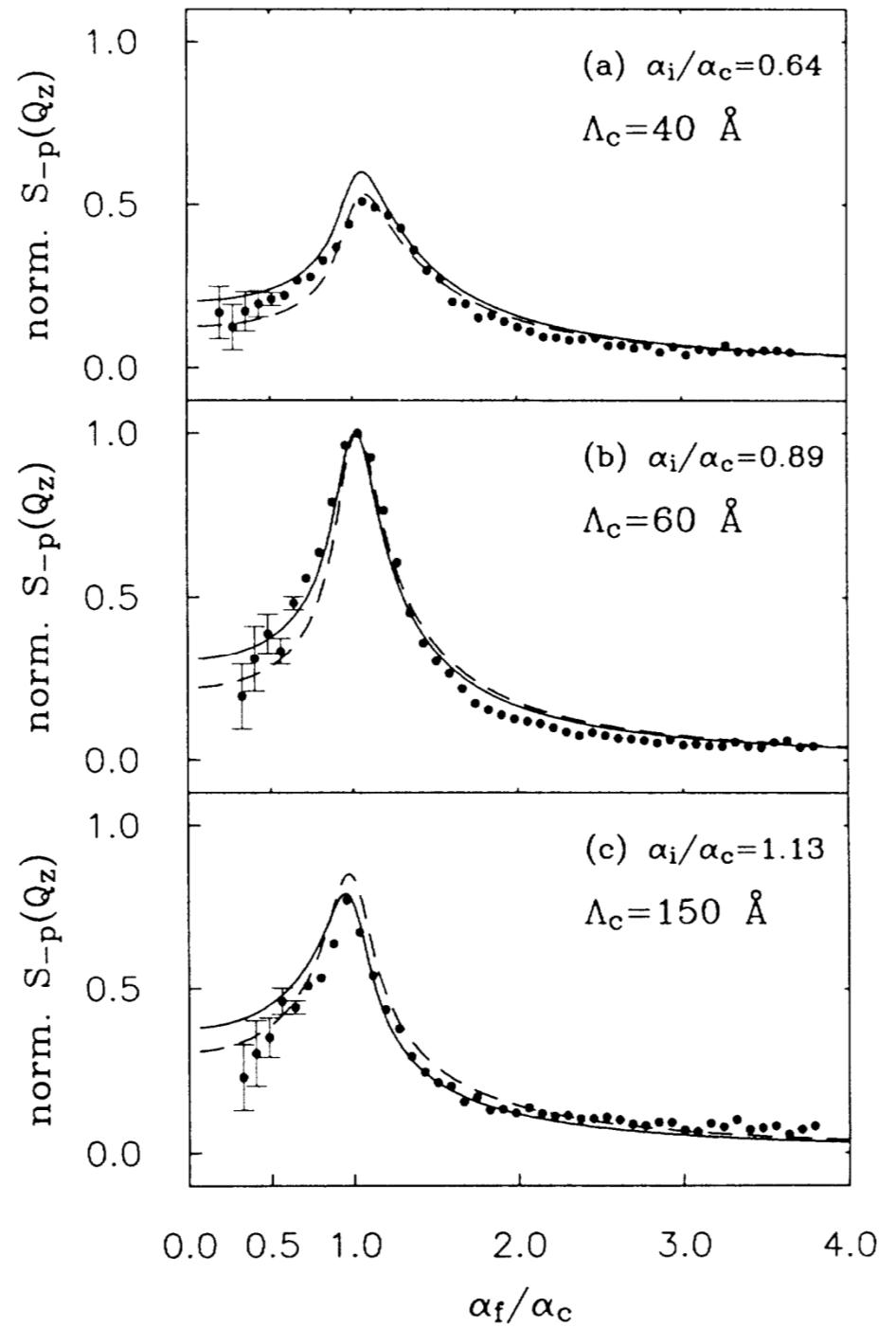
H. Dosch, PRB 35, 2137 (1987)

Distorted Wave Born Approximation

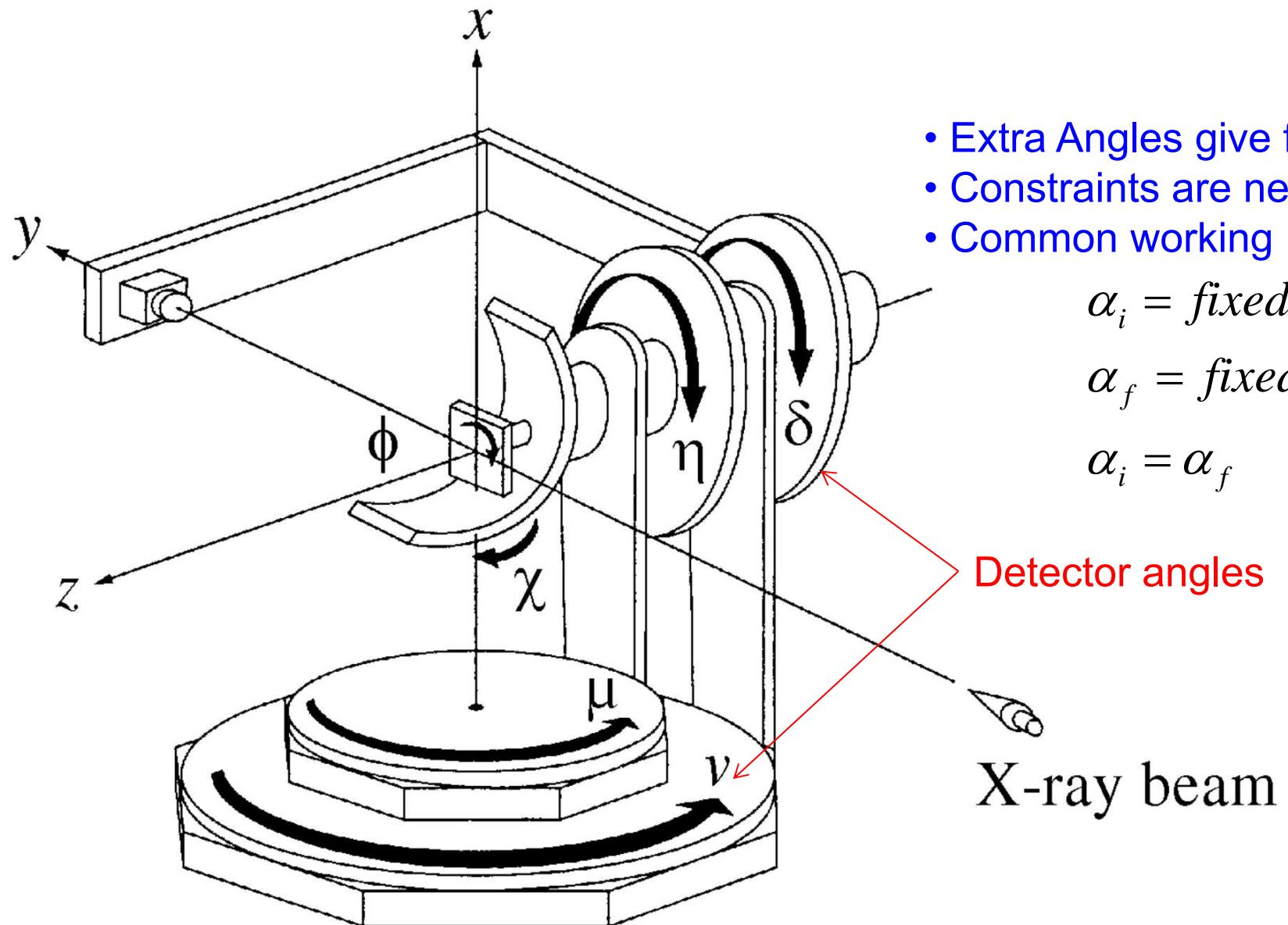
$$I_{\text{GID}}(Q') \propto |T_i|^2 S(Q') |T_f|^2$$

- refraction
- transmission

$S(\vec{Q}')$ is the structure factor using the internal wavevector transfer



A Six-Circle Diffractometer



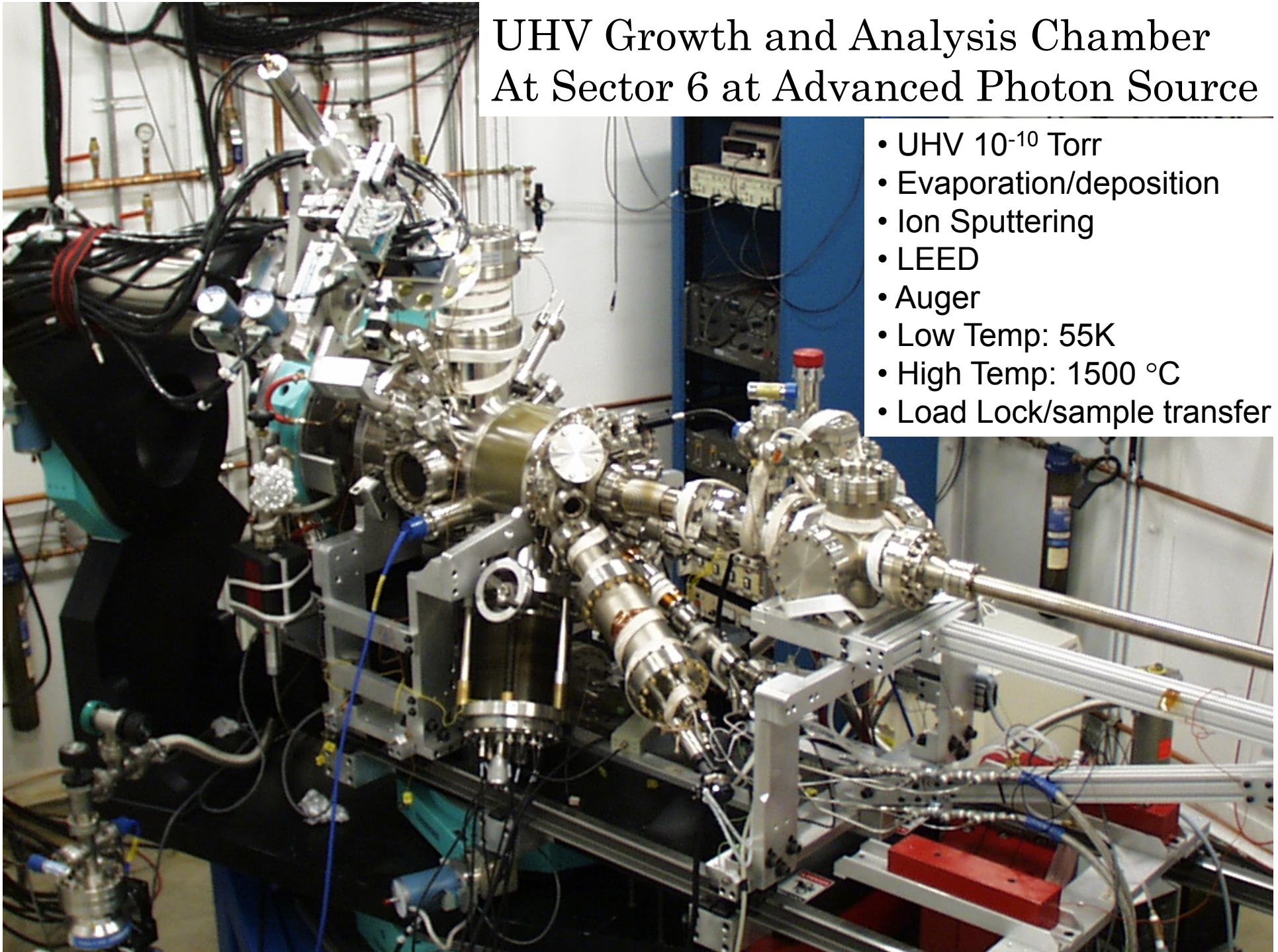
- Extra Angles give flexibility
- Constraints are needed
- Common working modes:

$\alpha_i = \text{fixed}$

$\alpha_f = \text{fixed}$

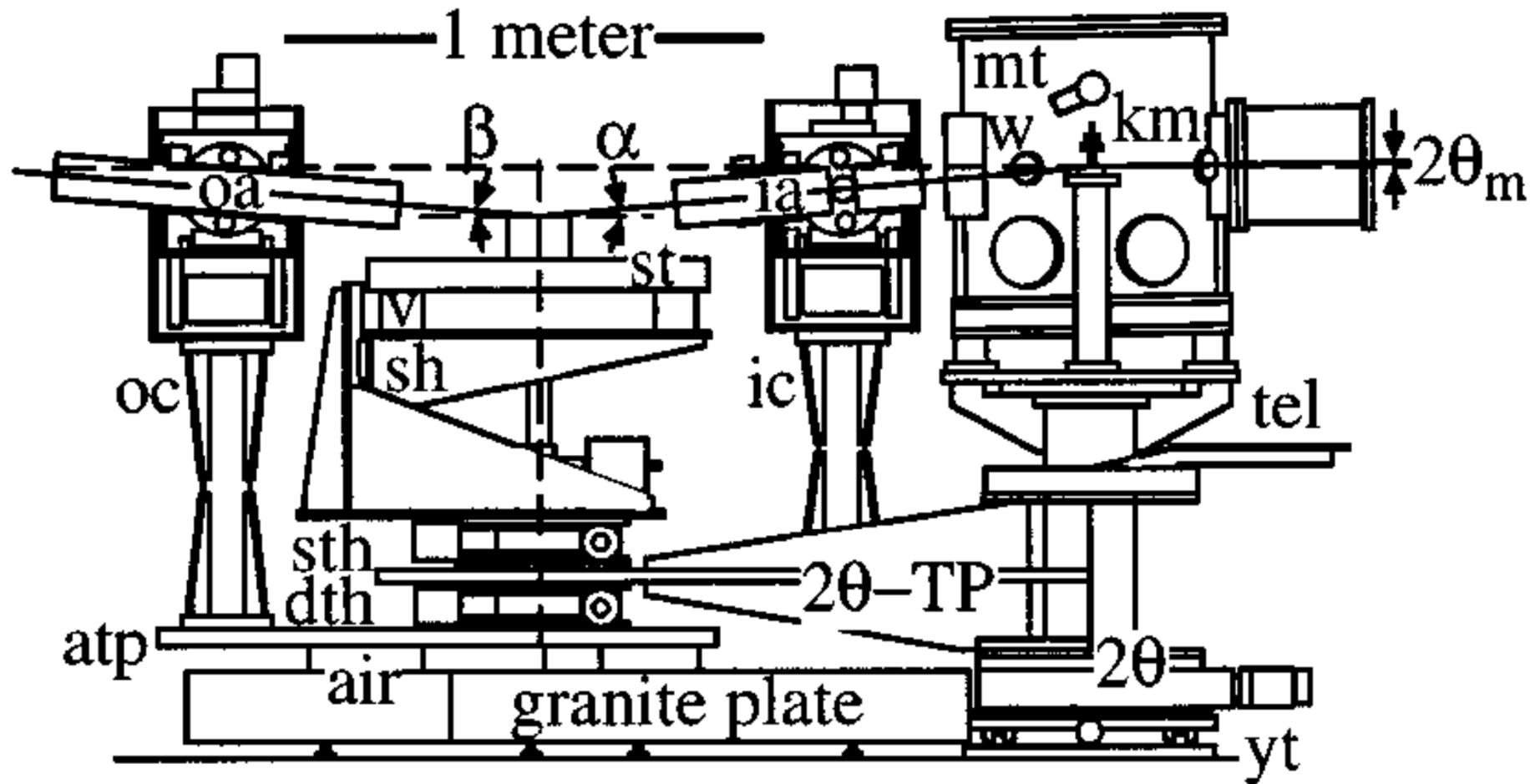
$\alpha_i = \alpha_f$

UHV Growth and Analysis Chamber At Sector 6 at Advanced Photon Source

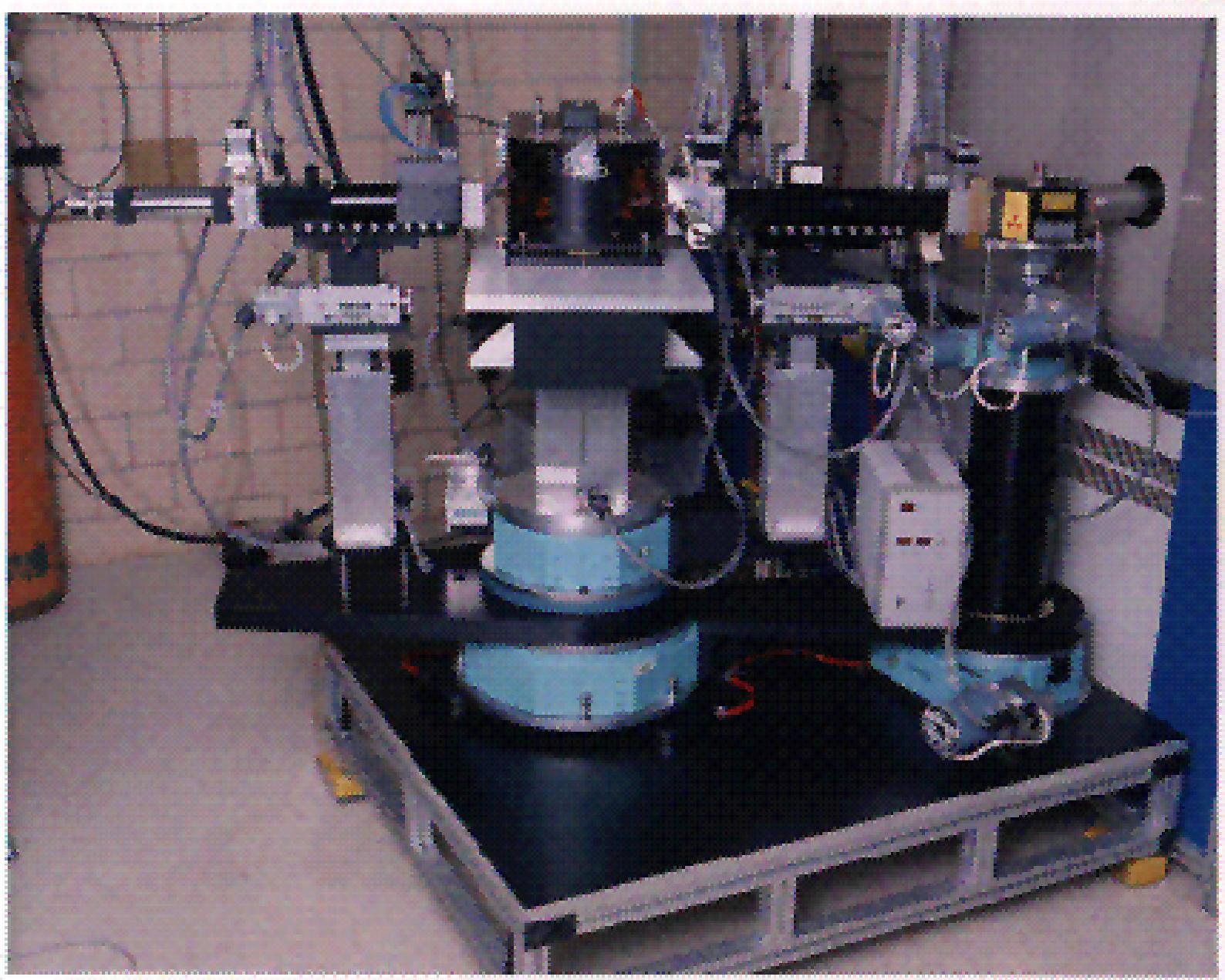


- UHV 10^{-10} Torr
- Evaporation/deposition
- Ion Sputtering
- LEED
- Auger
- Low Temp: 55K
- High Temp: 1500 °C
- Load Lock/sample transfer

Liquid Surface Diffractometer



M. Schlossman et. al., Rev. Sci. Inst. **68**, 4372 (1997)



David Vaknin, Ames Lab

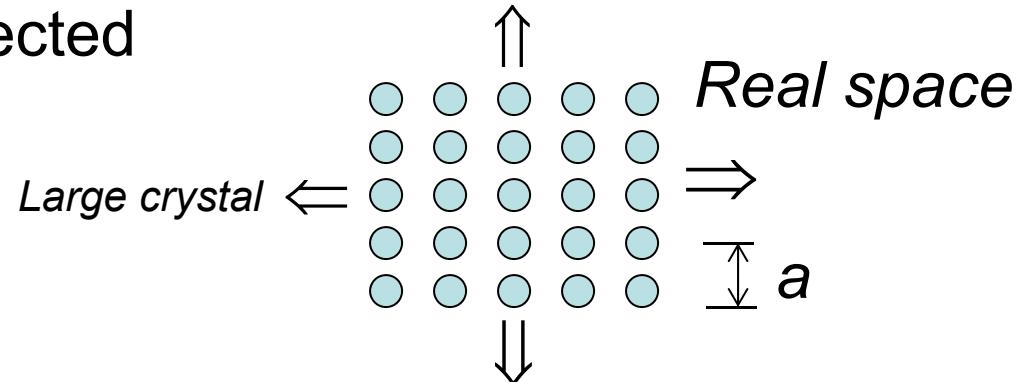
The Effect of a Crystalline Boundary



What is a crystal truncation rod?

First consider:

- Large crystals; rough and irregular boundaries
- Boundaries neglected



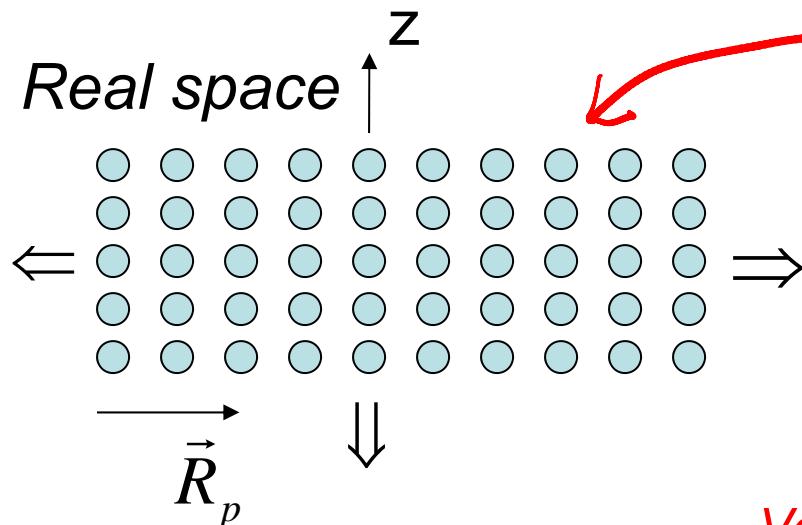
$$S(\vec{Q}) = \left| \sum_{n_x=0}^{N-1} e^{iQ_x a n_x} \sum_{n_y=0}^{N-1} e^{iQ_y b n_y} \sum_{n_z=0}^{N-1} e^{iQ_z c n_z} \right|^2 = \frac{\sin^2\left(\frac{NQ_x a}{2}\right)}{\sin^2\left(\frac{Q_x a}{2}\right)} \frac{\sin^2\left(\frac{NQ_y b}{2}\right)}{\sin^2\left(\frac{Q_y b}{2}\right)} \frac{\sin^2\left(\frac{NQ_z c}{2}\right)}{\sin^2\left(\frac{Q_z c}{2}\right)}$$

$$\propto \sum_{\vec{G}} \delta(\vec{Q} - \vec{G})$$



\vec{G} is a reciprocal lattice vector

Now consider a crystal with one atomically flat boundary...



- Large crystal; flat boundary $z = 0$
- Neglect boundary at $z = -\infty$

$$S(\vec{Q}) = b^2 \left| \sum_{n_z=0}^{N-1} \varepsilon^{n_z} e^{-iQ_z c n_z} \right|^2 \sum_{\vec{R}_p} \sum_{\vec{R}'_p} e^{i\vec{Q}_p \cdot (\vec{R}_p - \vec{R}'_p)}$$

Very small attenuation $\varepsilon < \sim 1$

$$\lim_{\varepsilon \rightarrow 1} \lim_{N \rightarrow \infty} \left| \frac{1 - \varepsilon^N e^{-iQ_z c N}}{1 - e^{-iQ_z c}} \right|^2 = \frac{1}{4 \sin^2 \left(\frac{Q_z c}{2} \right)} \approx \frac{1}{c^2 (Q_z - G_z)^2}$$

Reflected wave vanishes

Crystal Truncation Rod Factor

By neglecting the lateral boundaries:

$$\sum_{\vec{R}_p} \sum_{\vec{R}'_p} e^{i\vec{Q}_p \cdot (\vec{R}_p - \vec{R}'_p)} = N_{irr} \sum_{\vec{R}_p} e^{i\vec{Q}_p \cdot \vec{R}_p}$$

and $\sum_{\vec{R}_p} e^{i\vec{Q}_p \cdot \vec{R}_p} = \frac{(2\pi)^2}{S_c} \sum_{\vec{G}_p} \delta(\vec{Q}_p - \vec{G}_p)$

N_{irr} = the number of irradiated atoms at the surface

S_c = area per surface atom

\vec{G}_p = an in-plane reciprocal lattice vector

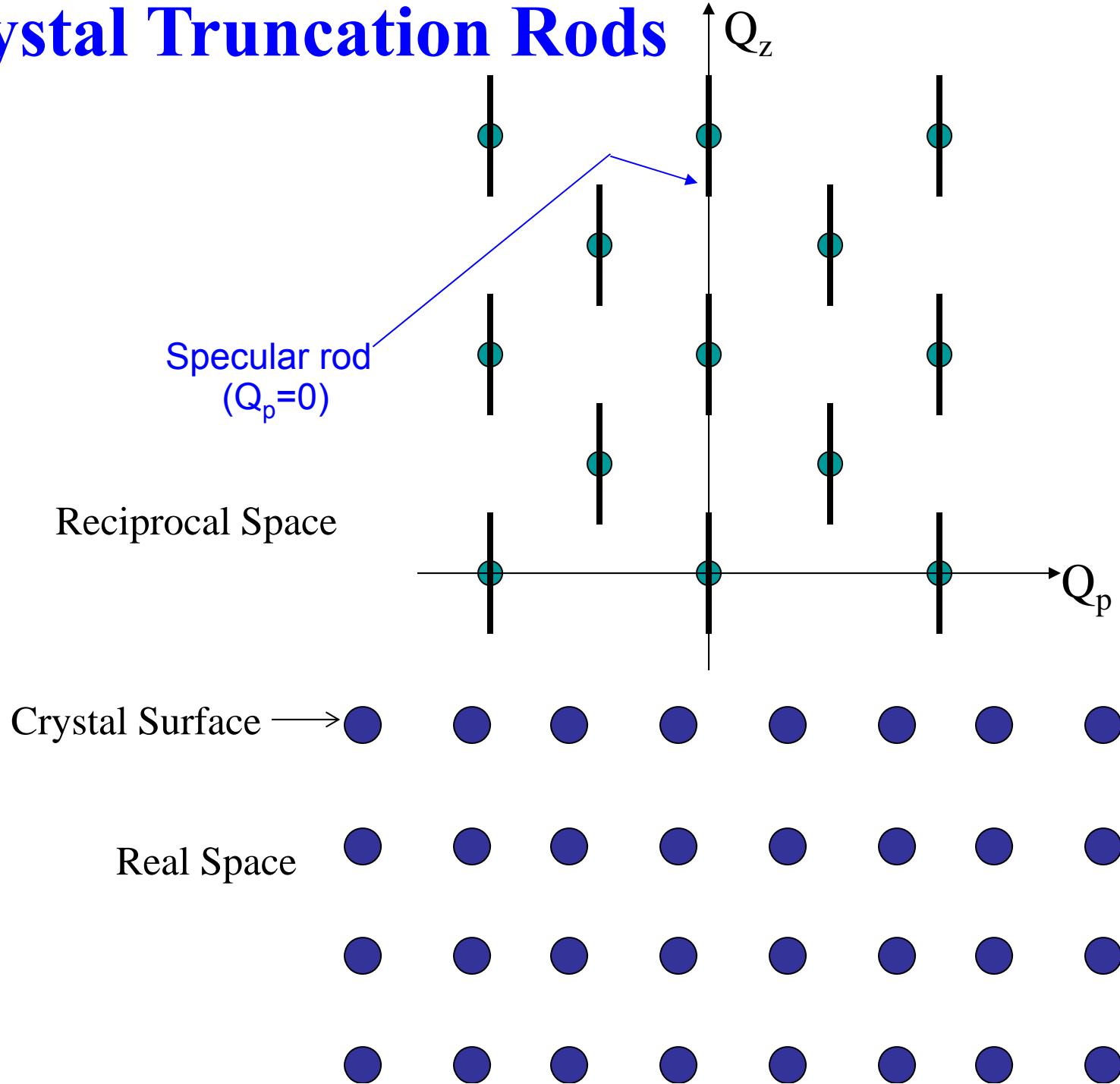
$$S(\vec{Q}) = \frac{(2\pi)^2 N_{irr} b^2}{S_c} \frac{1}{4 \sin^2\left(\frac{Q_z c}{2}\right)} \sum_{\vec{G}_p} \delta(\vec{Q}_p - \vec{G}_p)$$

CTR

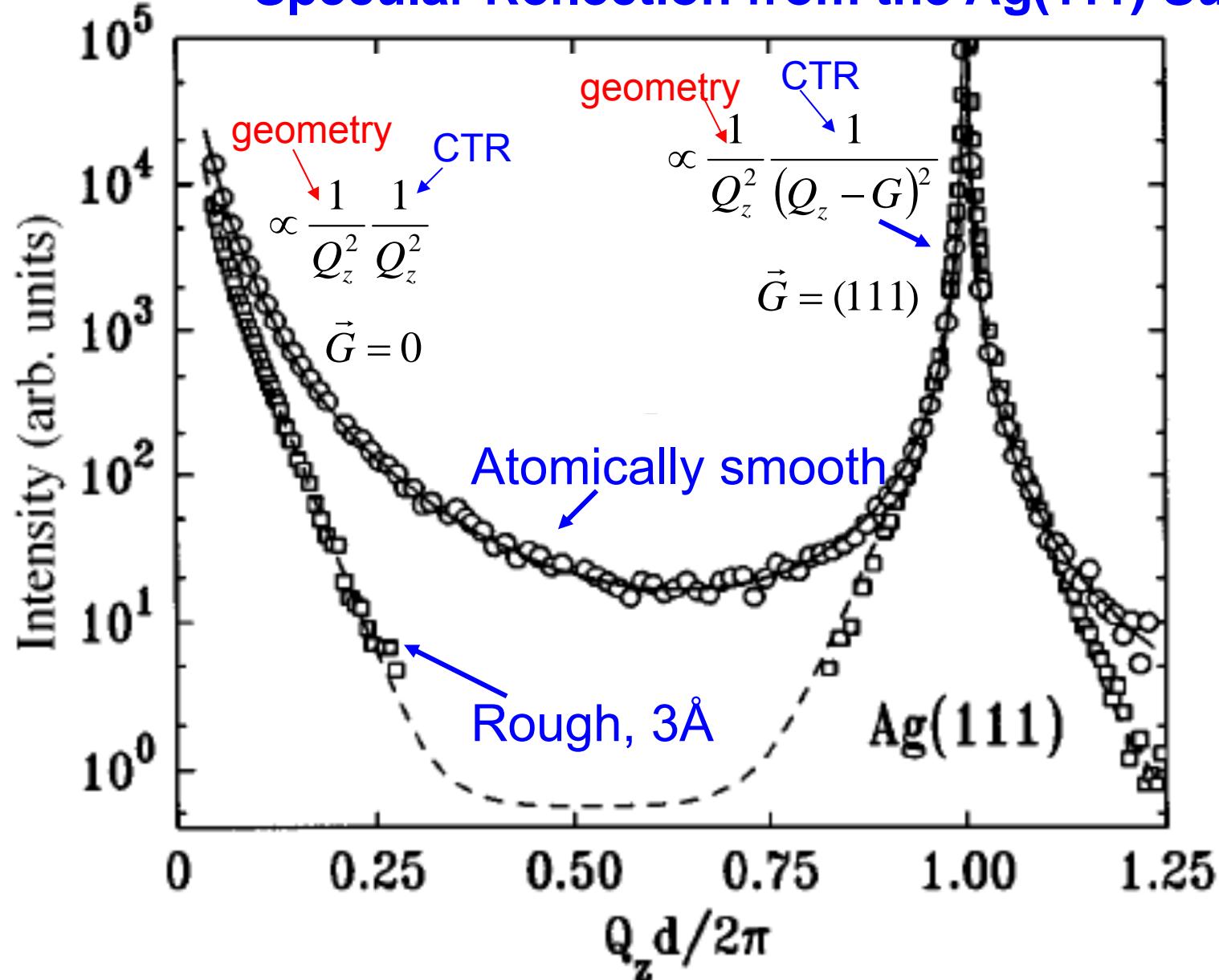
Narrow reflection in-plane

Intensity falls slowly

Crystal Truncation Rods

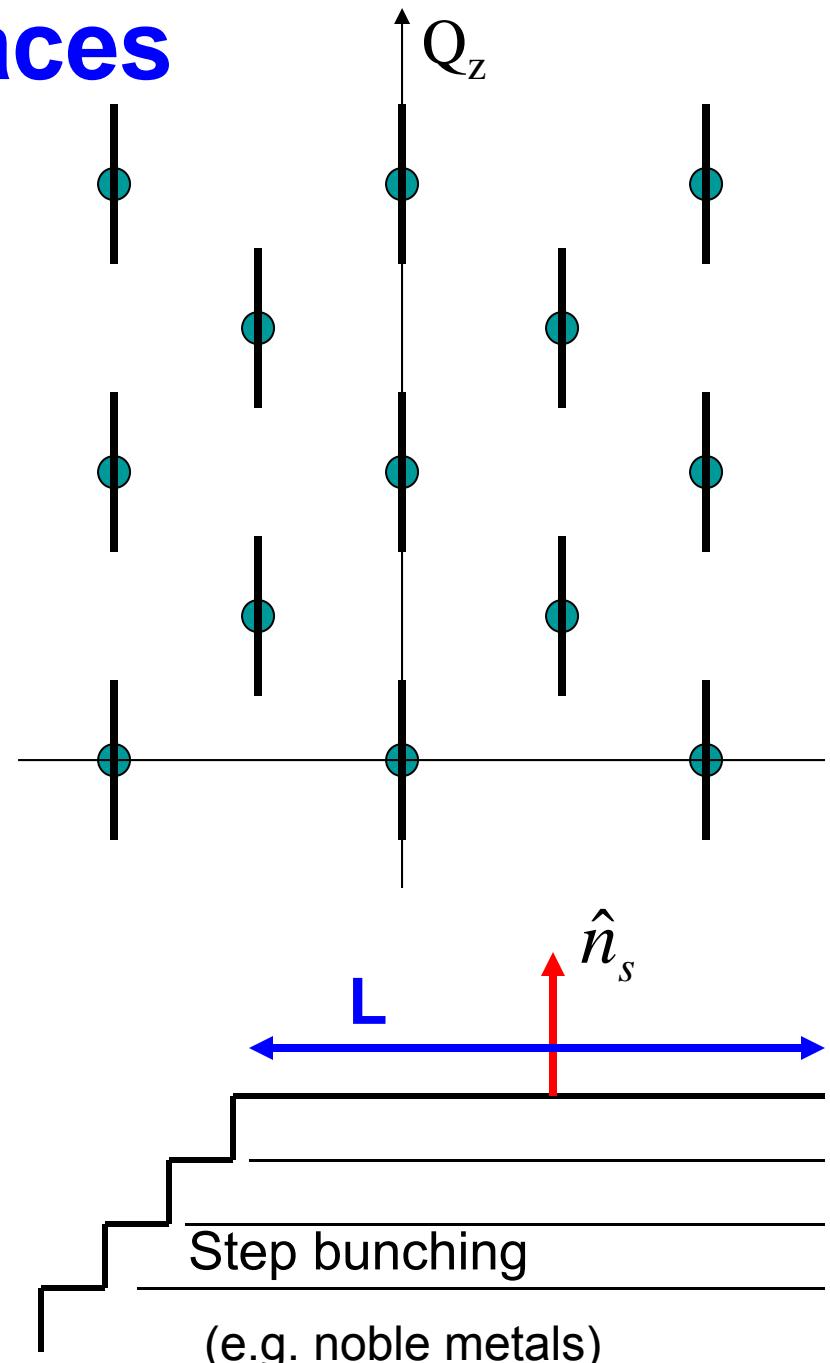
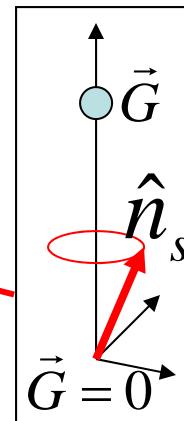
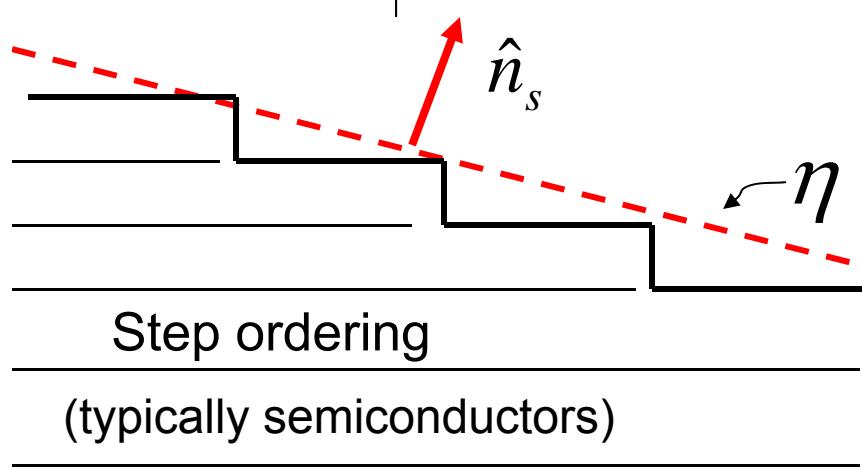
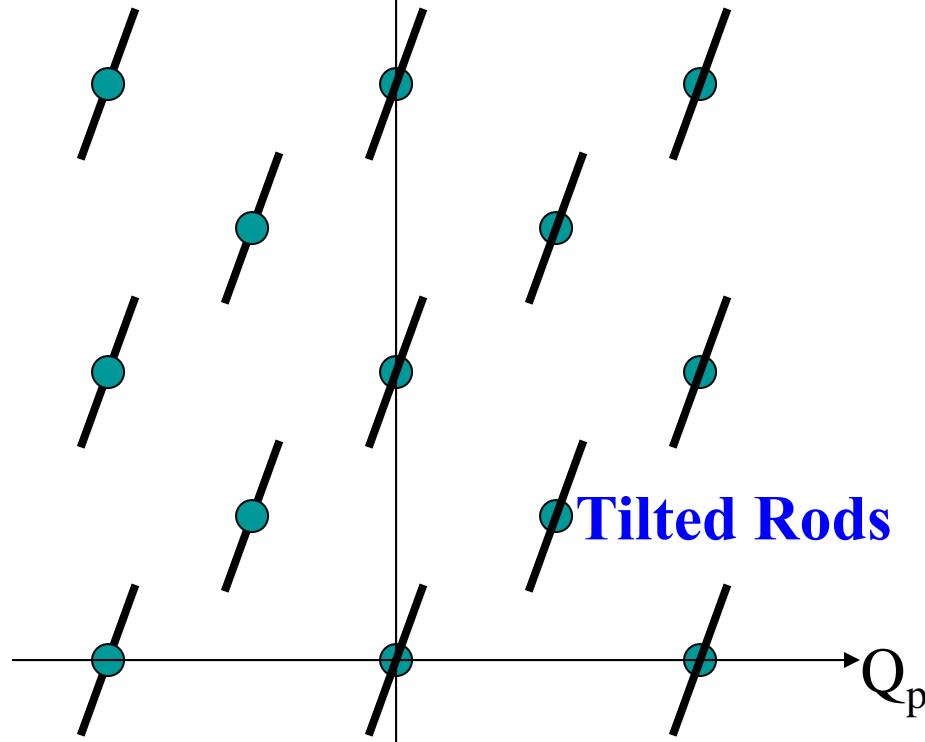


Crystal Truncation Rod Scattering for Specular Reflection from the Ag(111) Surface

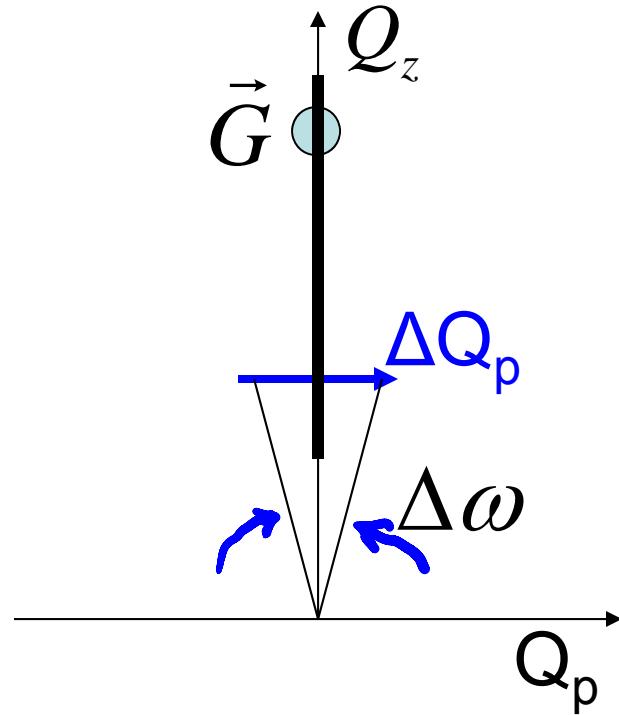
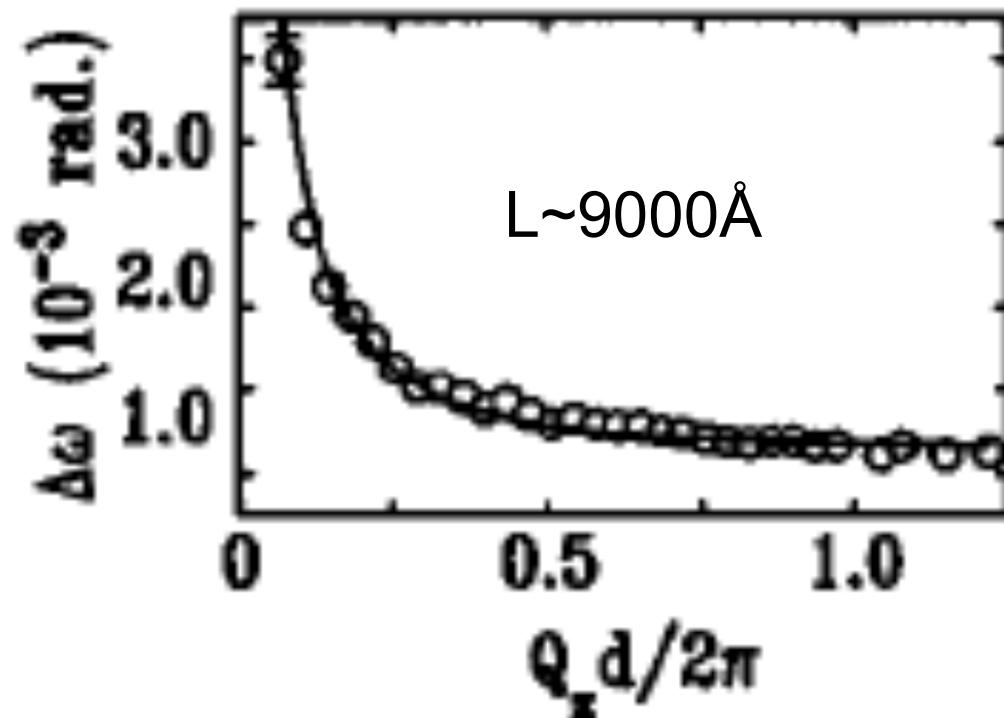
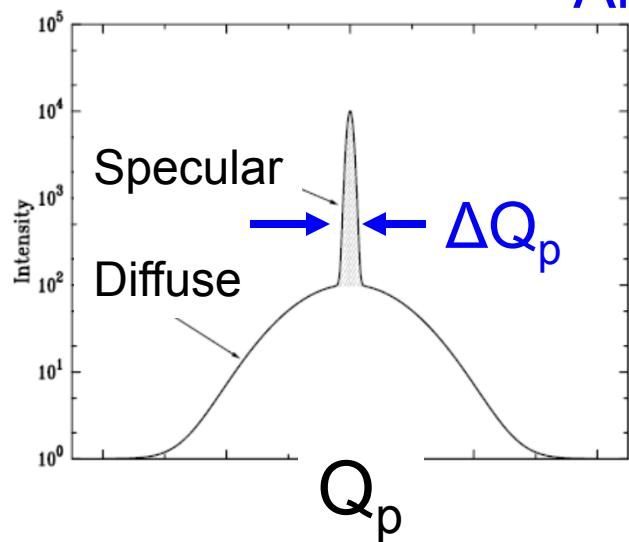


Elliott et. al. PRB 54, 17938 (1996)

Miscut Surfaces



Angular Width of Specular Decreases with Q_z

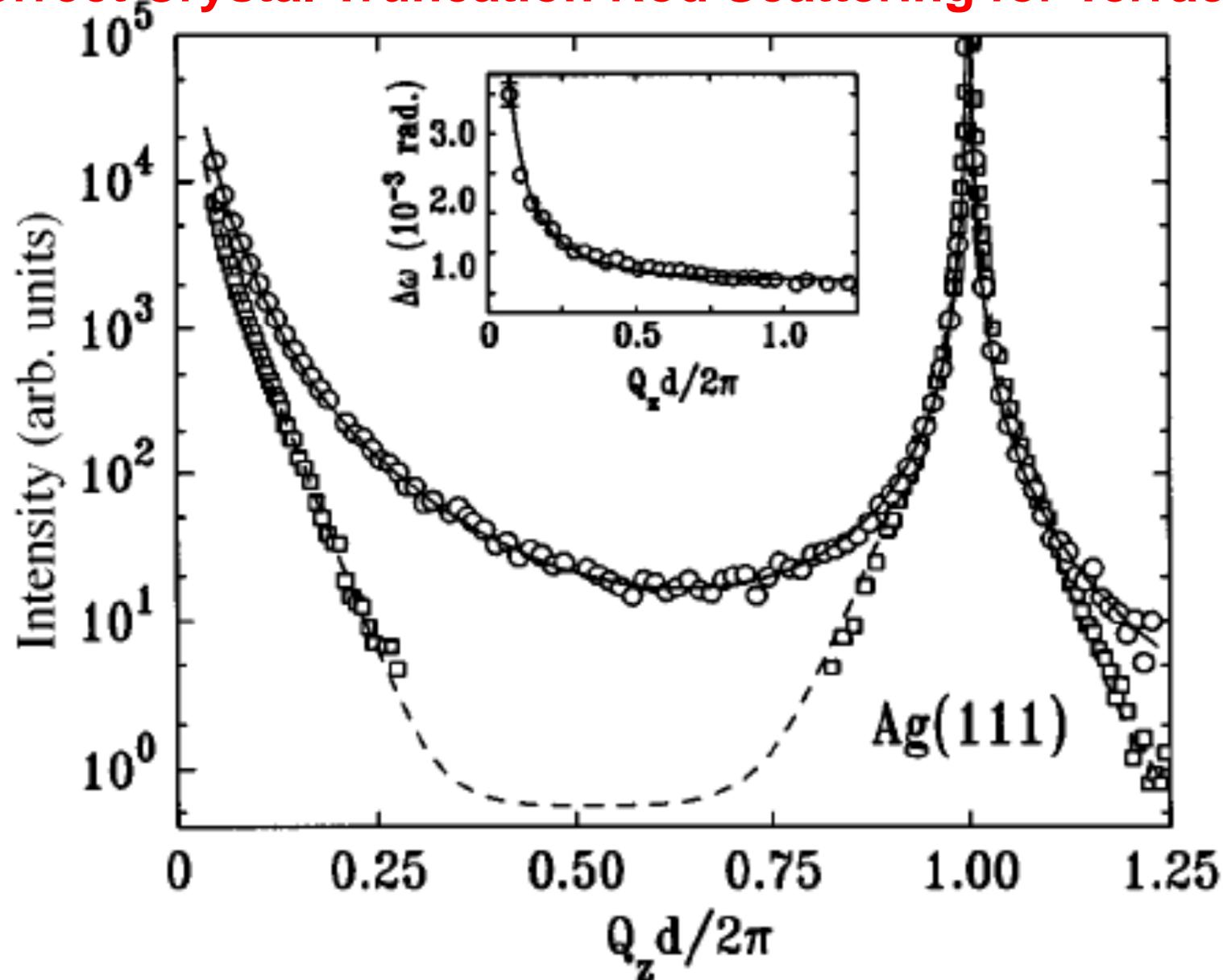


$$\Delta\omega = \frac{\Delta Q_p}{Q_z}$$

$$\Delta Q_p = \frac{2\pi}{L}$$

Can determine terrace size, L

Specular Reflection from the Ag(111) Surface
Correct Crystal Truncation Rod Scattering for Terrace Size

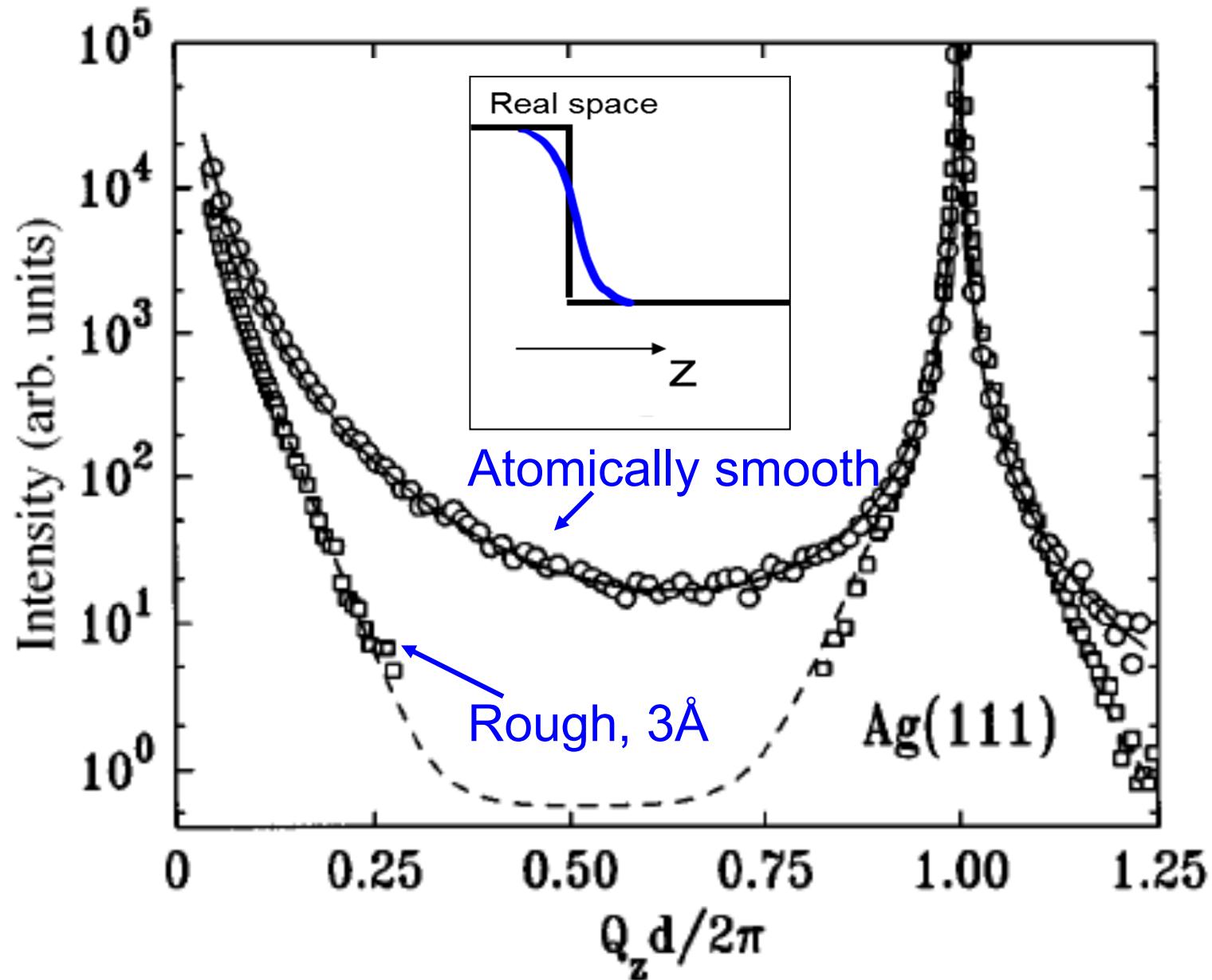


Elliott et. al. PRB 54, 17938 (1996)

The Effect of a Rough Surface



Sharper interface (real space) gives “broader” scattering



Elliott et. al. PRB 54, 17938 (1996)

Special location along CTR: anti-Bragg

Bragg Position:

$$Q_z c = 2\pi m$$

$$I = \left| \sum_{n_z=0}^{N-1} e^{-i2\pi n_z} \right|^2 \rightarrow N^2$$

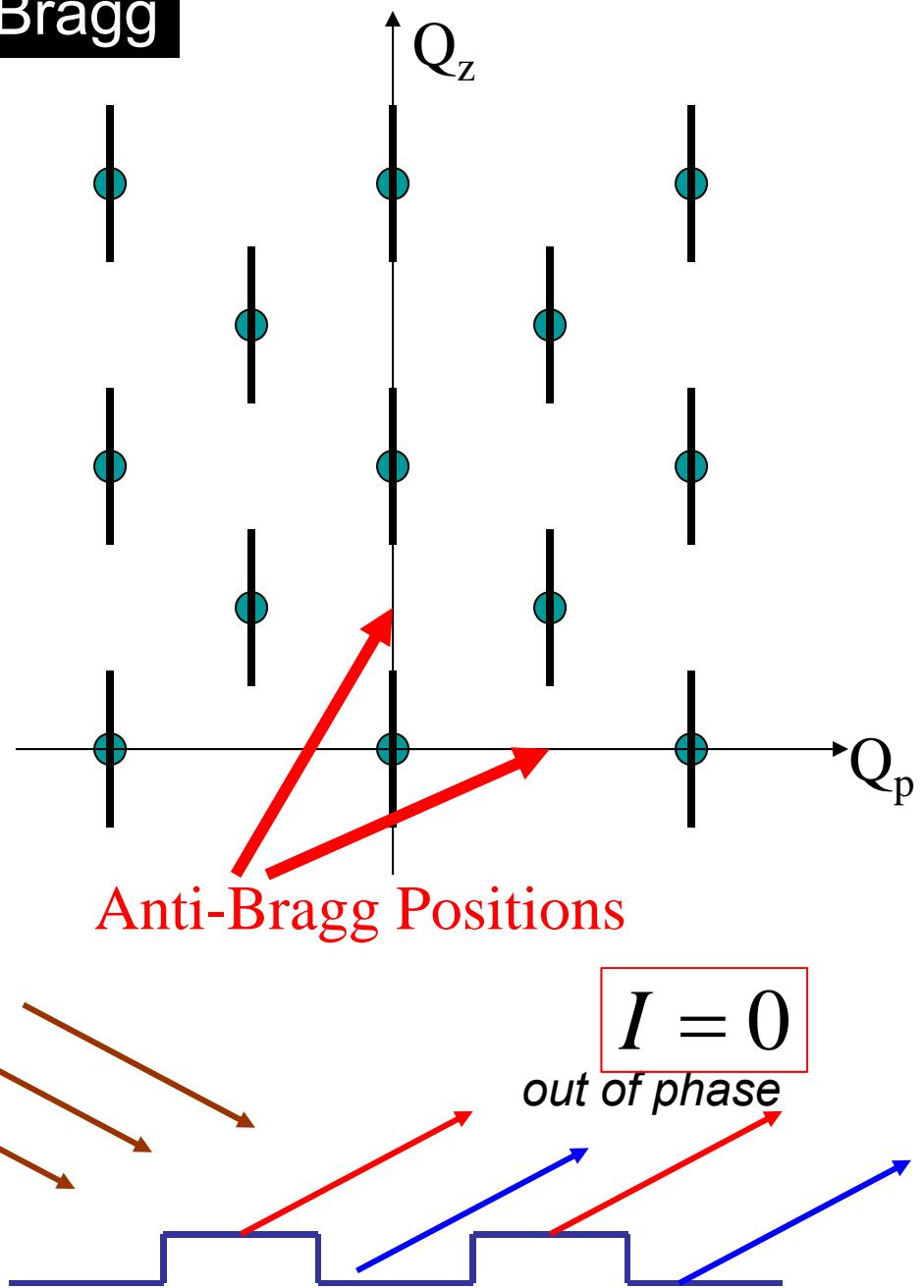
Anti-Bragg Position:

$$Q_z c = \pi m \quad (m \text{ odd})$$

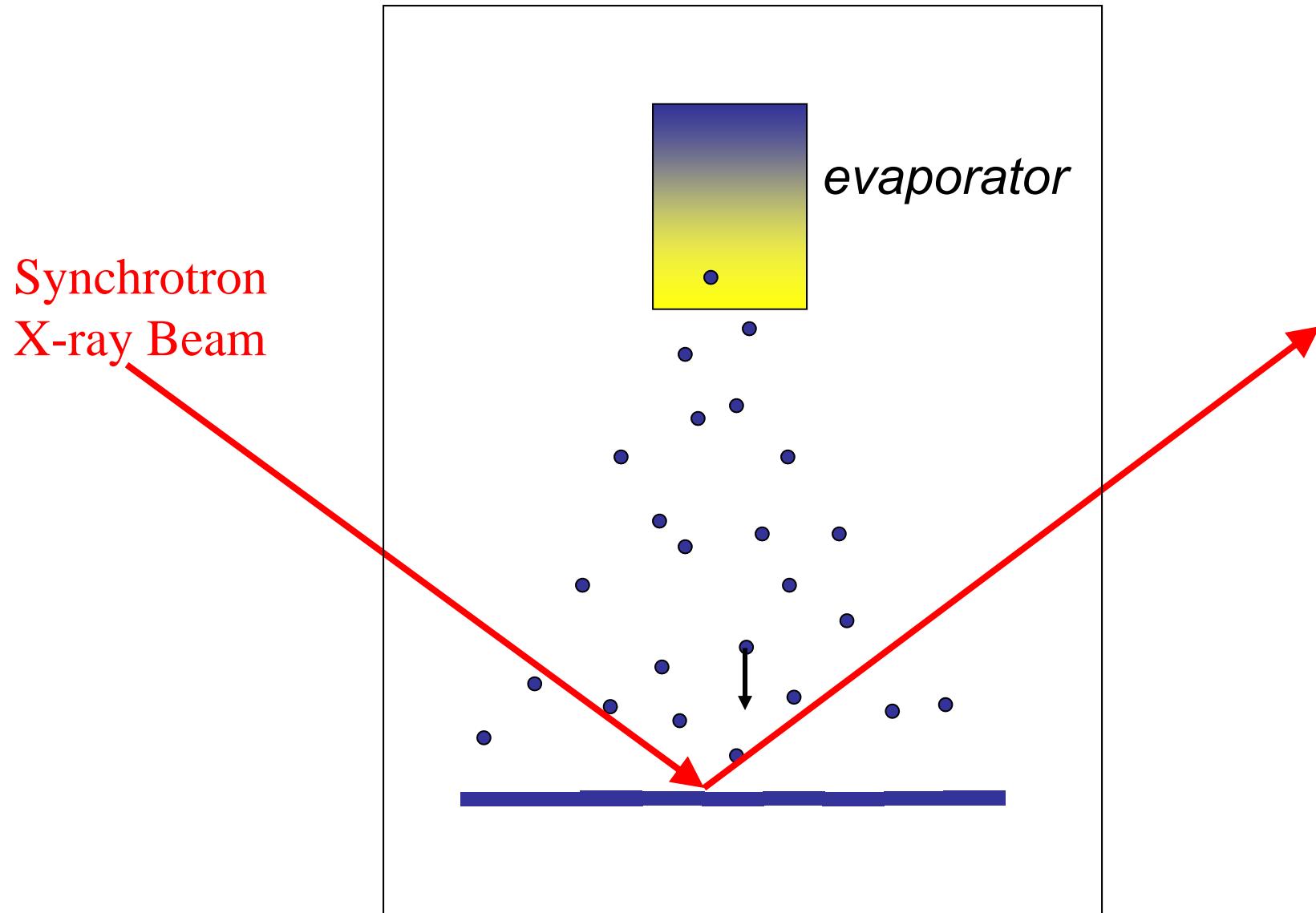
$$I = \left| \sum_{n_z=0}^{N-1} e^{-i\pi n_z} \right|^2 \rightarrow 1$$

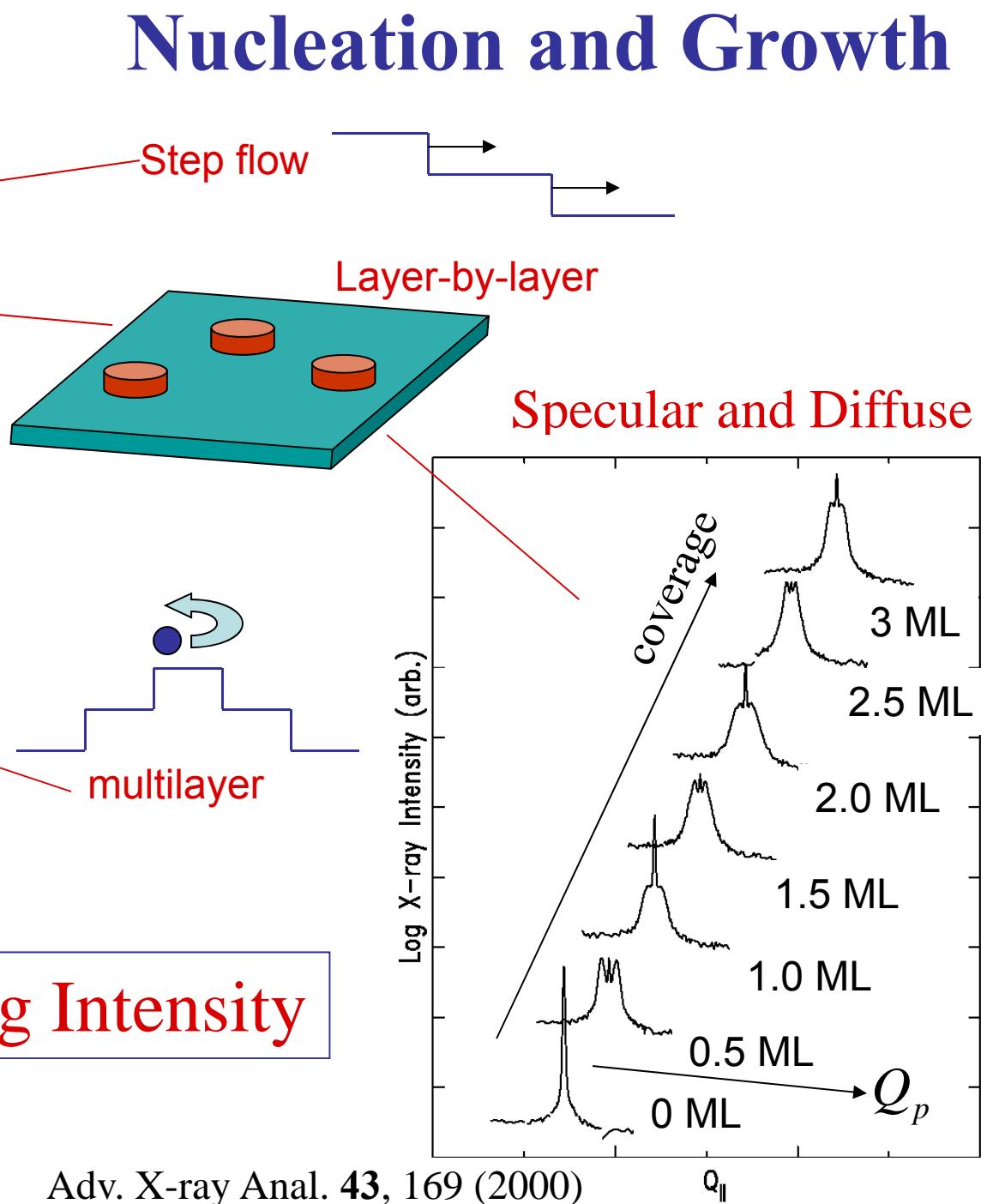
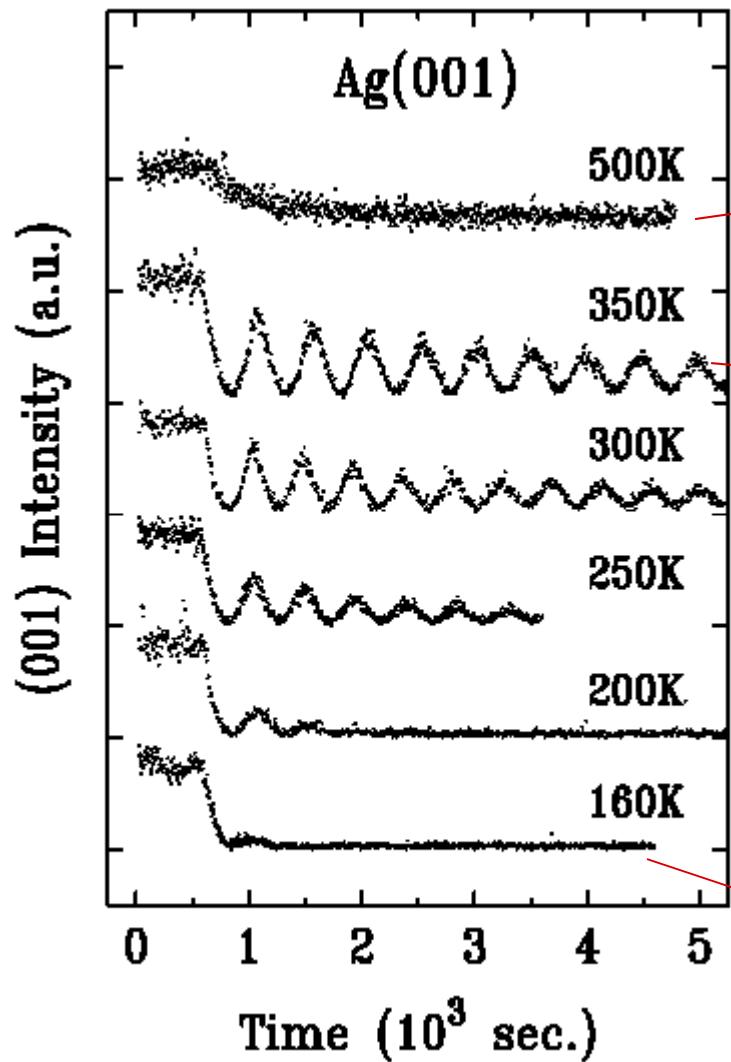
For a smooth surface

1 atomic layer



In situ vapor deposition in UHV





Specular Anti-Bragg Intensity

Diffuse Scattering

Caused by lateral structure

$$|A(\vec{Q})|^2 =$$

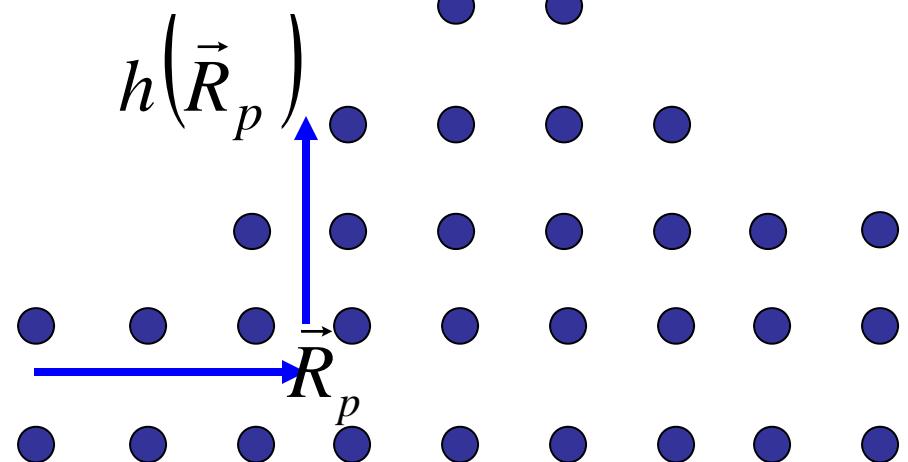
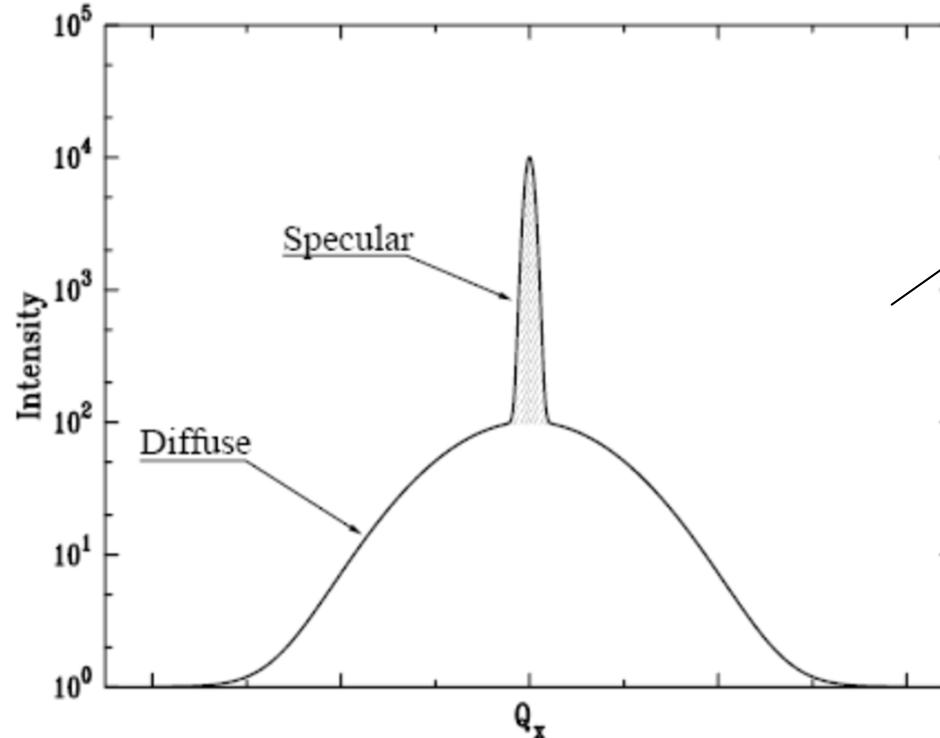
$$= |b|^2 \sum_{\vec{R}_p} \sum_{\vec{R}'_p}$$

$$= \frac{N_{irr} |b|^2}{1 - e^{iQ_z c}}$$

CTR factor

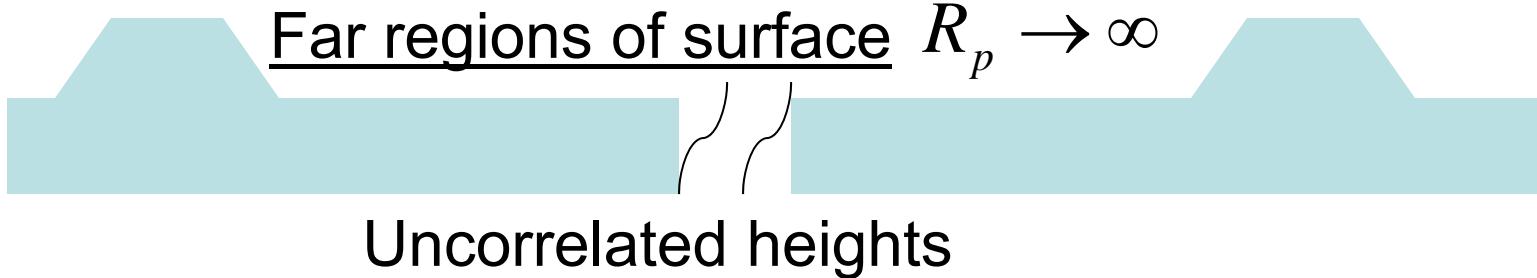
FT of average phase difference
due to lateral height-differences

- Neglect lateral boundaries



Q_p dependence
 $= S_T(Q_p)$

Transverse Lineshape



$$\left\langle e^{iQ_z(h(\vec{R}_p + \vec{R}'_p) - h(\vec{R}'_p))} \right\rangle_{\vec{R}'_p} \rightarrow \left| \left\langle e^{iQ_z h} \right\rangle \right|^2$$

Uncorrelated Roughness @ Large Distance Gives Bragg:

$$S_T^{Bragg}(\vec{Q}_p) = \frac{(2\pi)^2}{S_c} \delta(\vec{Q}_p - \vec{G}_p) \left| \left\langle e^{iQ_z h} \right\rangle \right|^2$$

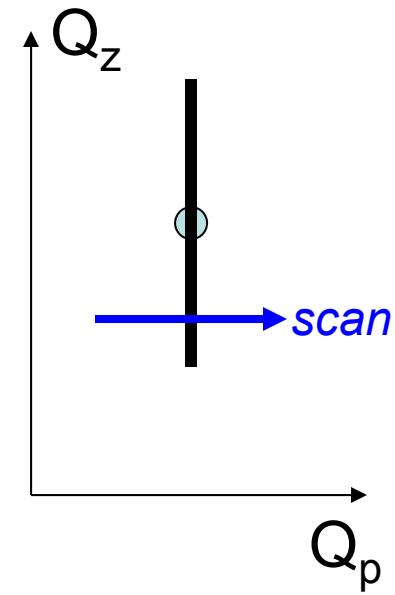
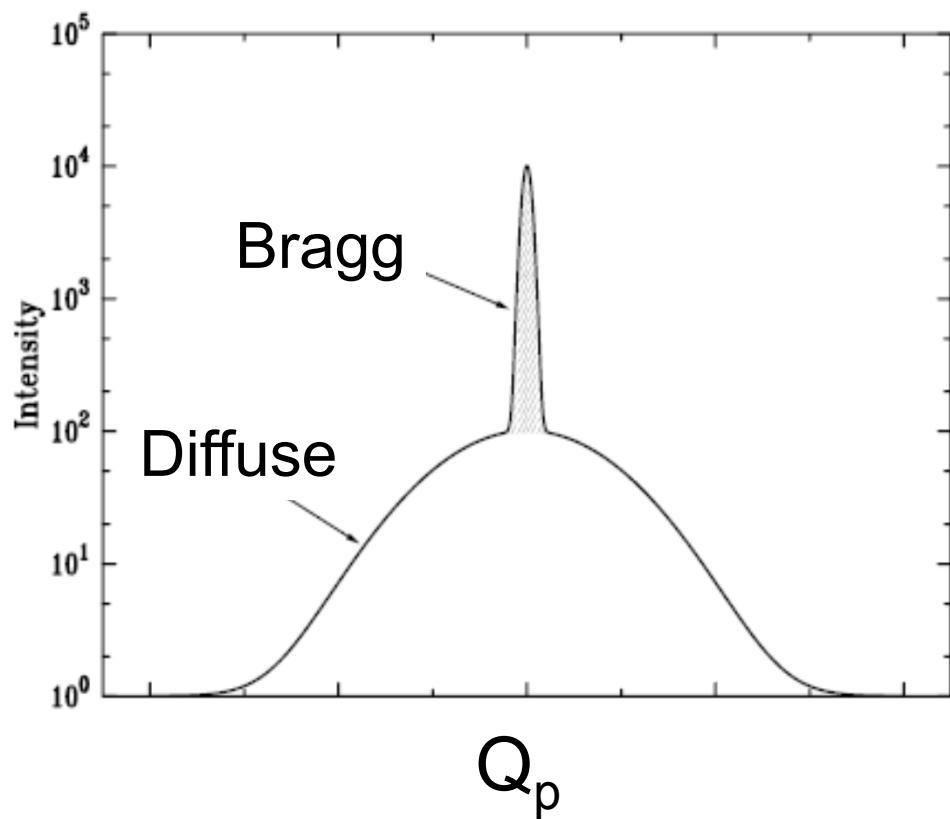
Short-Range Correlations Give Diffuse Scattering:

$$S_T^{Diffuse}(\vec{Q}_p) = \sum_{\vec{R}_p} e^{i\vec{Q}_p \cdot \vec{R}_p} \left\{ \left\langle e^{iQ_z(h(\vec{R}_p + \vec{R}'_p) - h(\vec{R}'_p))} \right\rangle_{\vec{R}'_p} - \left| \left\langle e^{iQ_z h} \right\rangle \right|^2 \right\}$$

Two Component Line Shape: Bragg + Diffuse

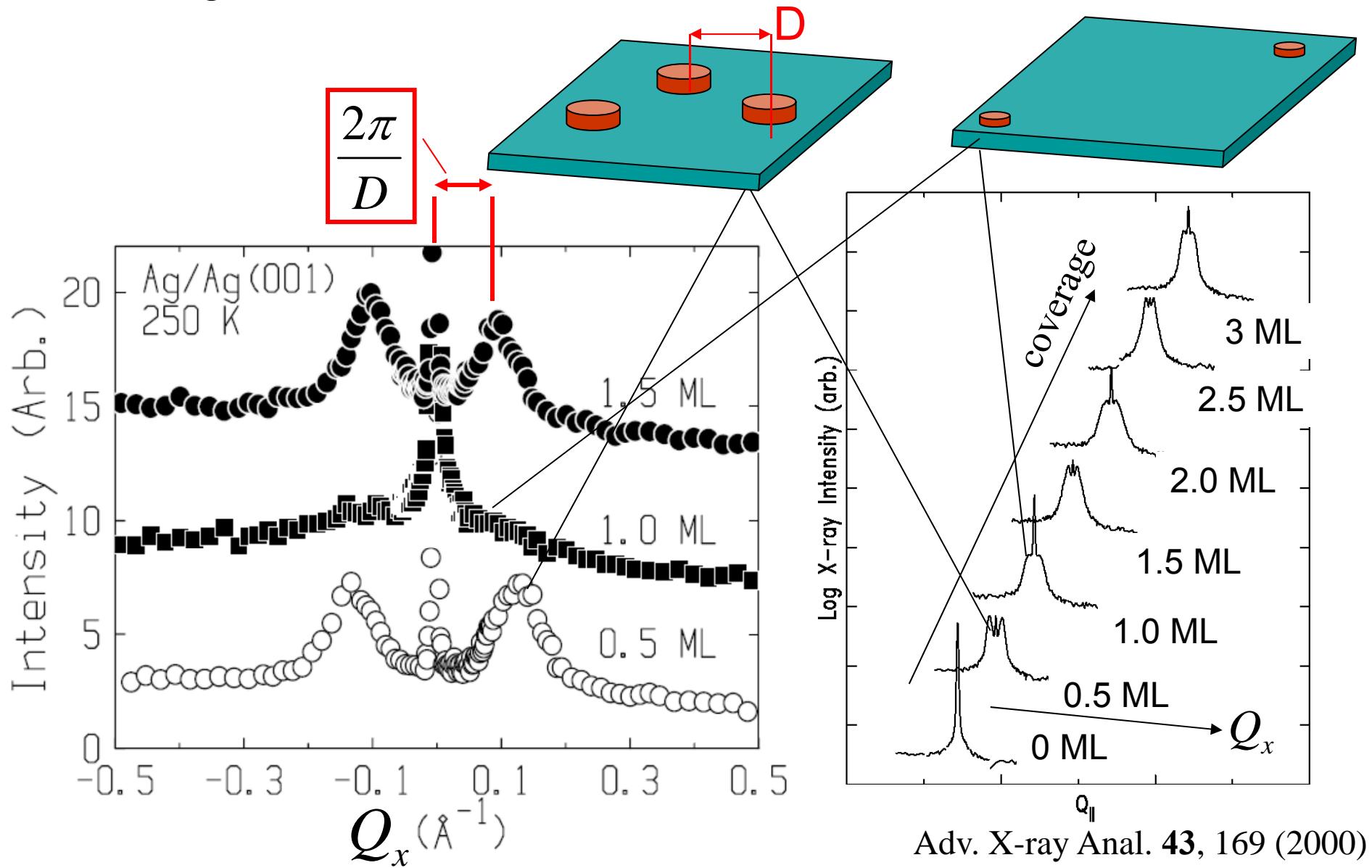
$$S_T(\vec{Q}_p) = S_T^{Bragg}(\vec{Q}_p) + S_T^{Diffuse}(\vec{Q}_p)$$

- Bragg due to laterally uncorrelated disorder at long distances
- Diffuse due to short-range correlations



Layer-by-layer growth

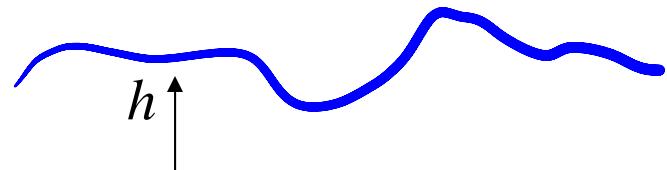
- Specular Bragg Rod: intensity changes with roughness
- Strong inter-island correlations seen in the diffuse



Attenuation of the Bragg Rod and Surface Roughness

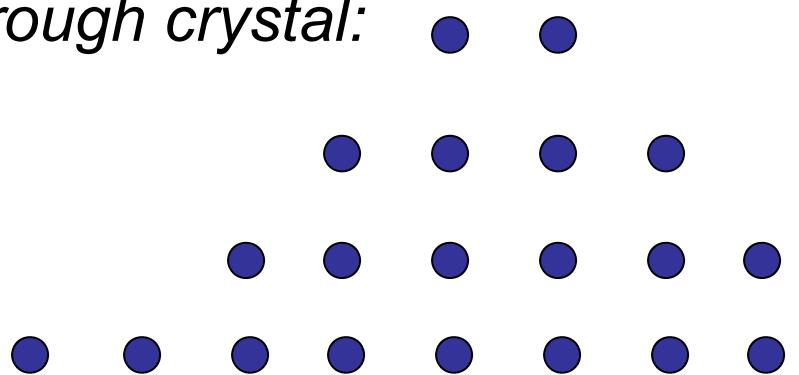
If height fluctuations are Gaussian: σ is rms roughness

$$\left| \langle e^{iQ_z h} \rangle \right|^2 \rightarrow e^{-Q_z^2 \sigma^2}$$



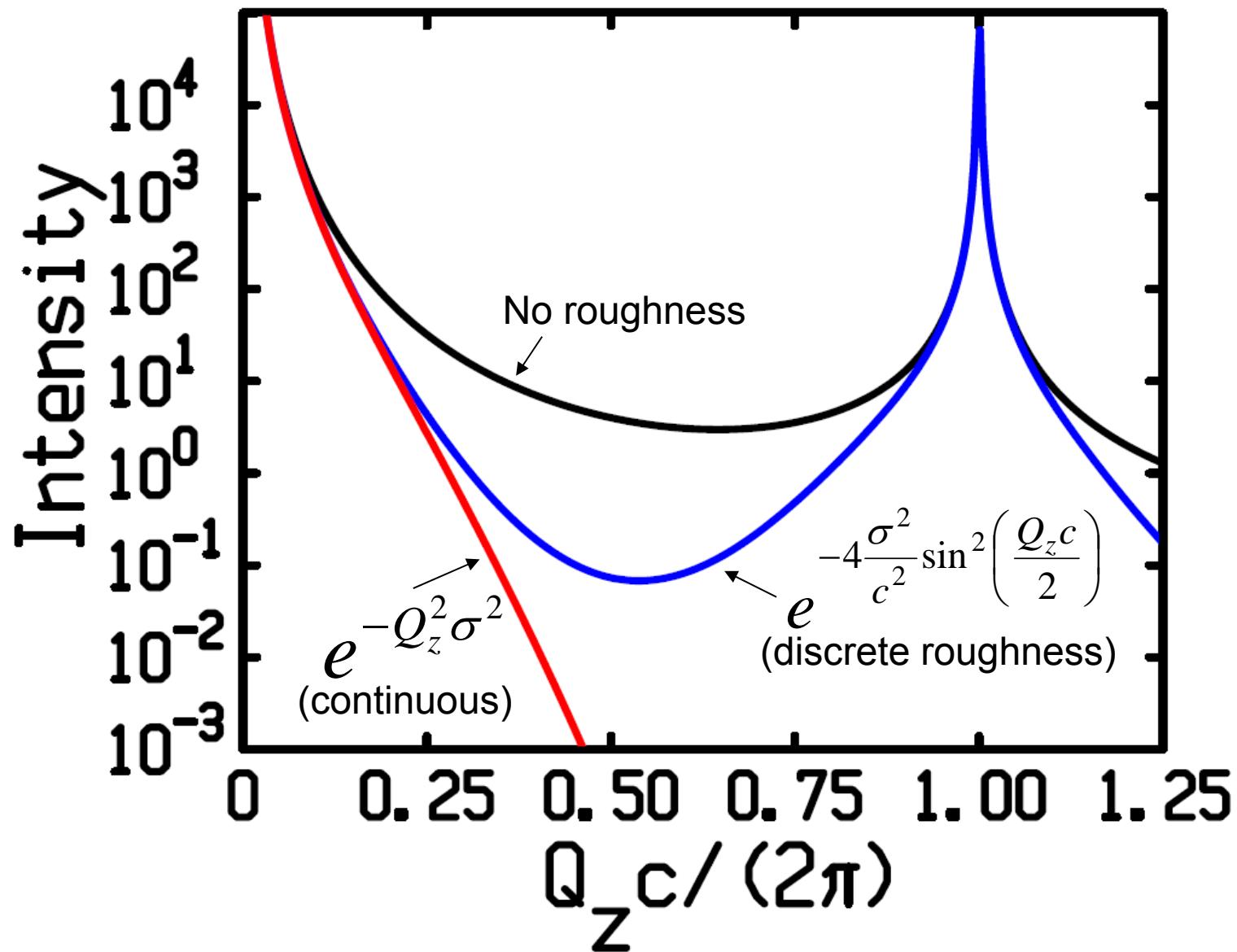
But crystal heights are discrete for a rough crystal:

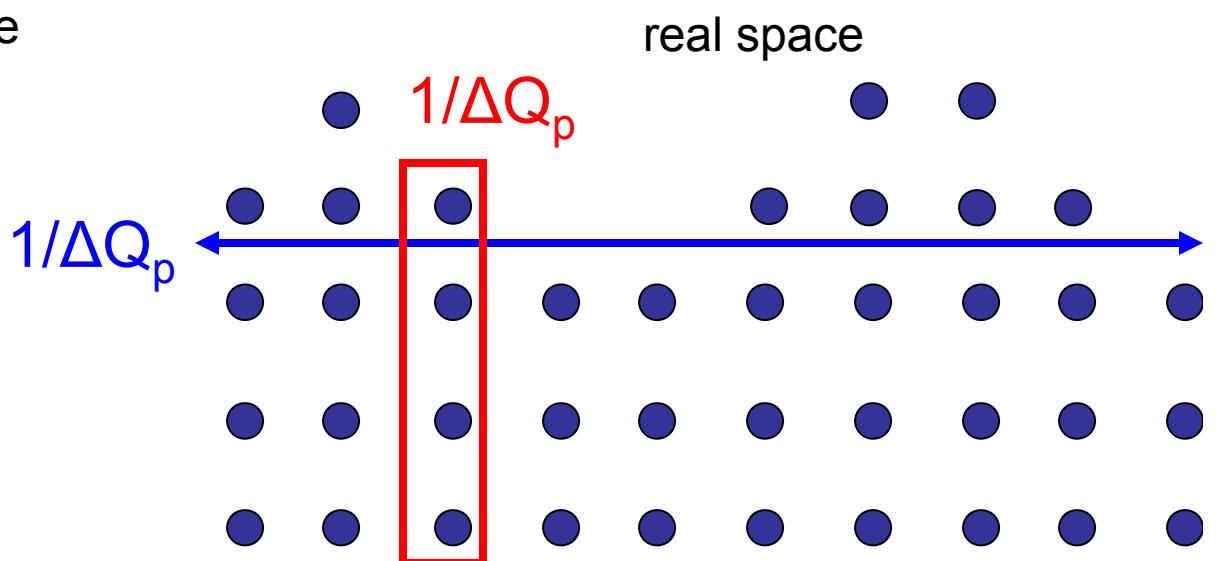
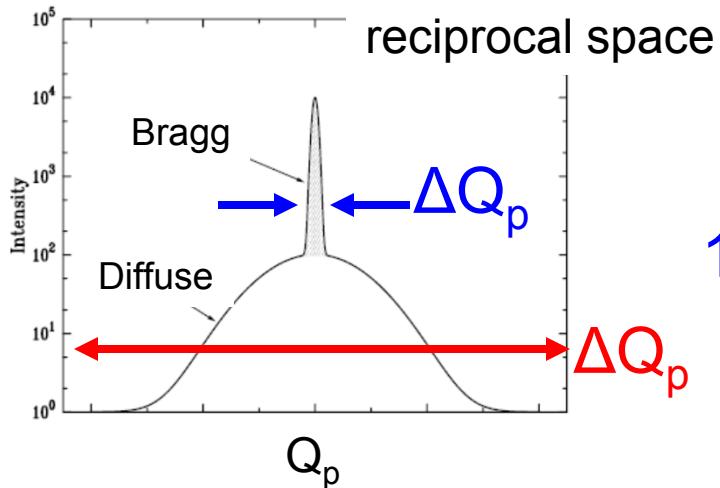
$$\left| \langle e^{iQ_z h} \rangle \right|^2 \rightarrow e^{-4\frac{\sigma^2}{c^2} \sin^2\left(\frac{Q_z c}{2}\right)}$$



- Binomial distribution (limits to a Gaussian for large roughness)
- Preserves translational symmetry in the roughness

- Sharper interface (real space) gives broader scattering
- Gaussian roughness does not give translational symmetry





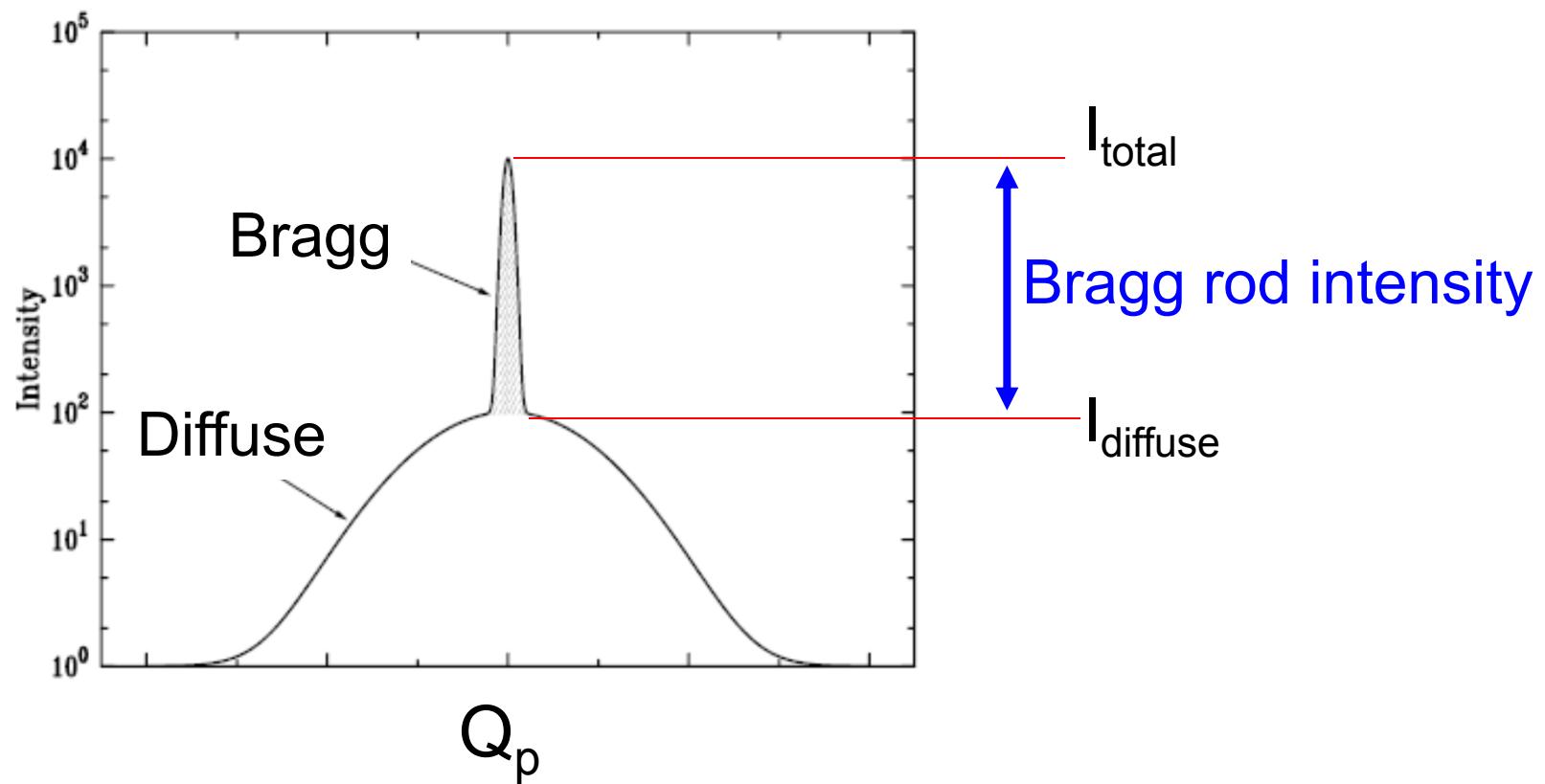
Bragg is narrow:
it samples laterally uncorrelated roughness at long distances

$$S^{Bragg}(\vec{Q}) \propto \frac{|b|^2}{|1 - e^{iQ_z c}|^2} e^{-4\frac{\sigma^2}{c^2} \sin^2\left(\frac{Q_z c}{2}\right)}$$

Transversely-integrated scattering shows no effect of roughness:

$$\iint d^2 Q_p S(\vec{Q}) \propto \frac{|b|^2}{|1 - e^{iQ_z c}|^2} \quad (for 1 interface)$$

In practice, at every Q_z the diffuse must be subtracted from the total intensity to get the Bragg rod intensity:

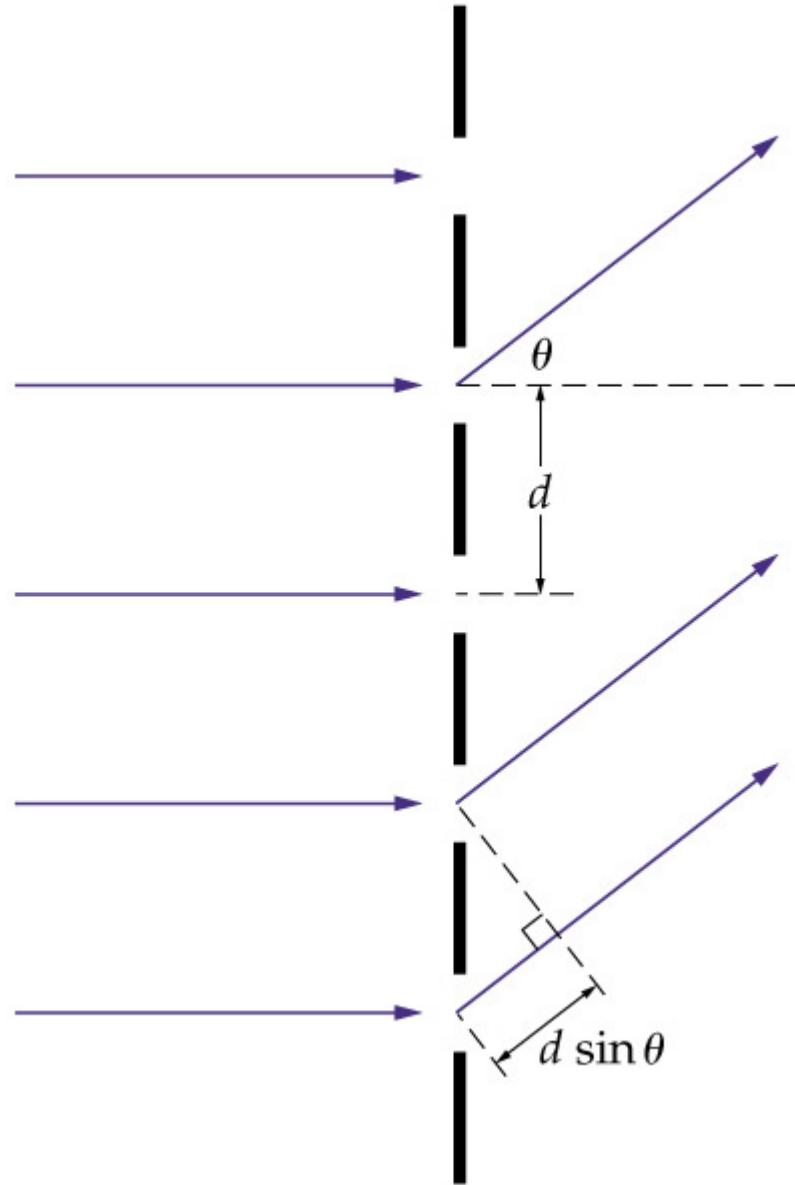


What do we expect from a Thin Film?

1st let's recall Young's slit interference... 

Recall...

N-Slit Interference and Diffraction Gratings

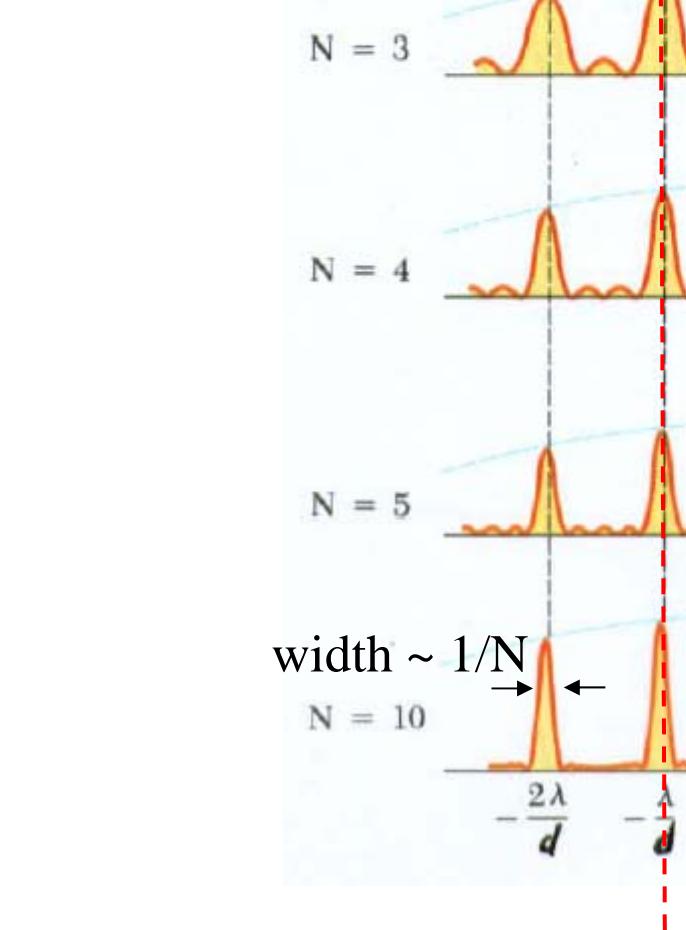


Principle maxima

$$d \sin \theta = m\lambda$$

Principle maxima always in the same place for N slits:
But they narrow: width $\sim 1/N$

Double Slit $N = 2$
(no subsidiary maxima)



Principle maxima
 $d \sin \theta = m\lambda$

($N-2$ subsidiary maxima)

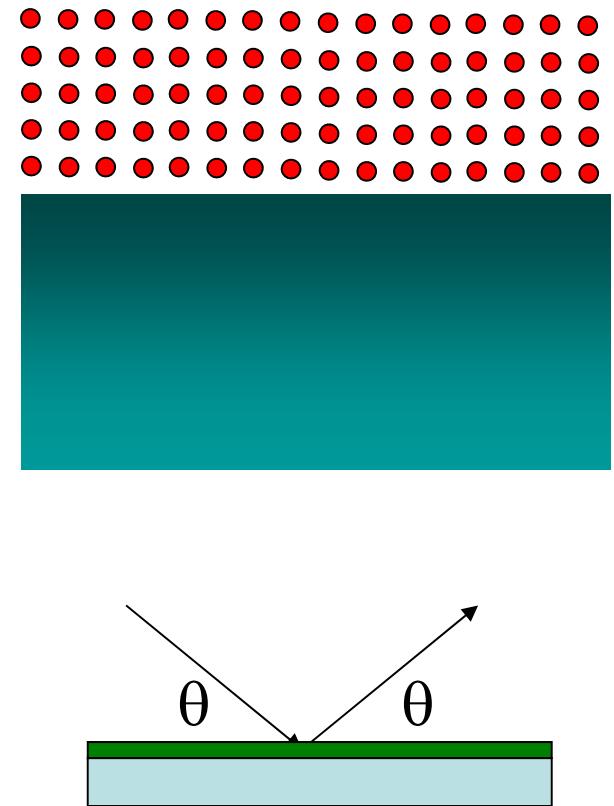
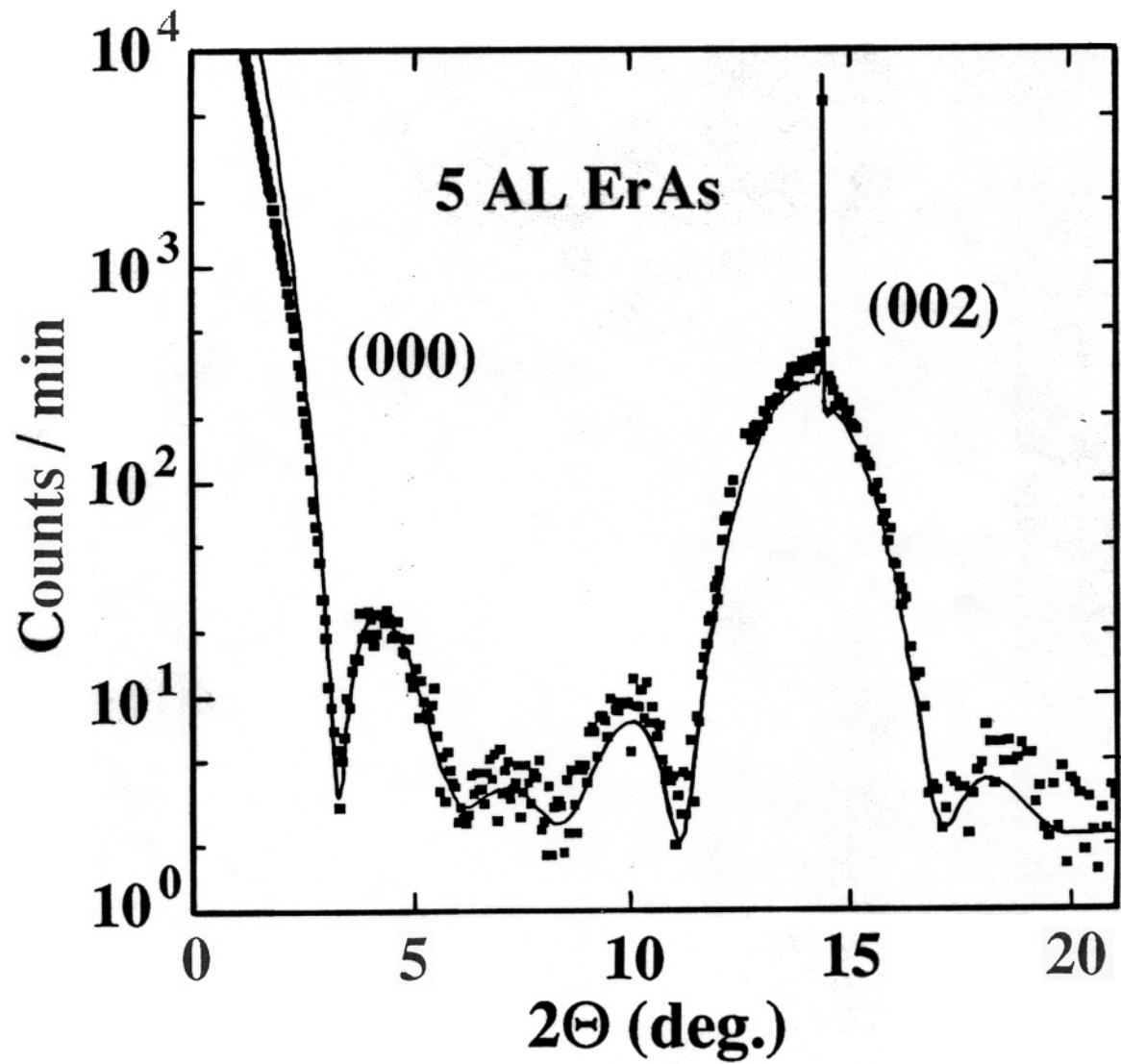
1 subsidiary maximum

2 subsidiary maximum

N large:

- Weak subsidiary maxima
- Sharp principle maxima

“5-slit” interference of x-rays from 5 layers of atoms

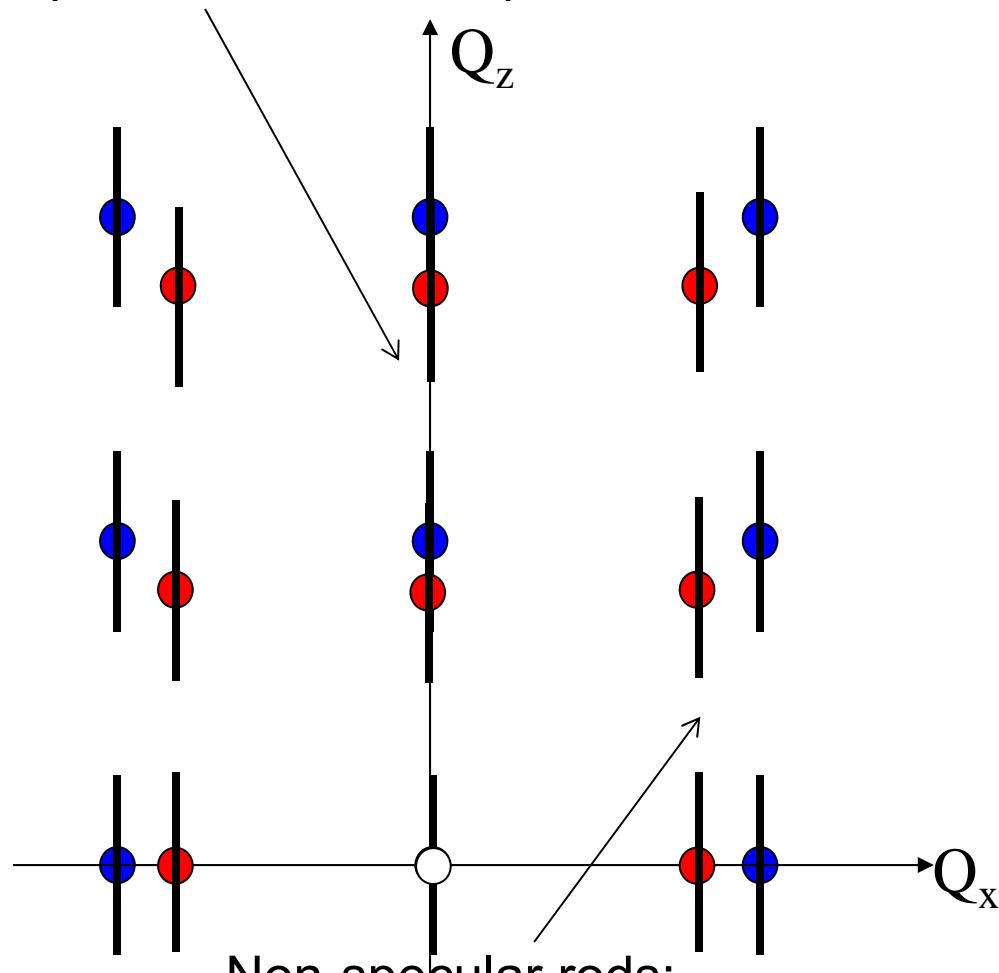


Miceli et al., Appl. Phys. Lett. **62**, 2060 (1992)

Thin Films

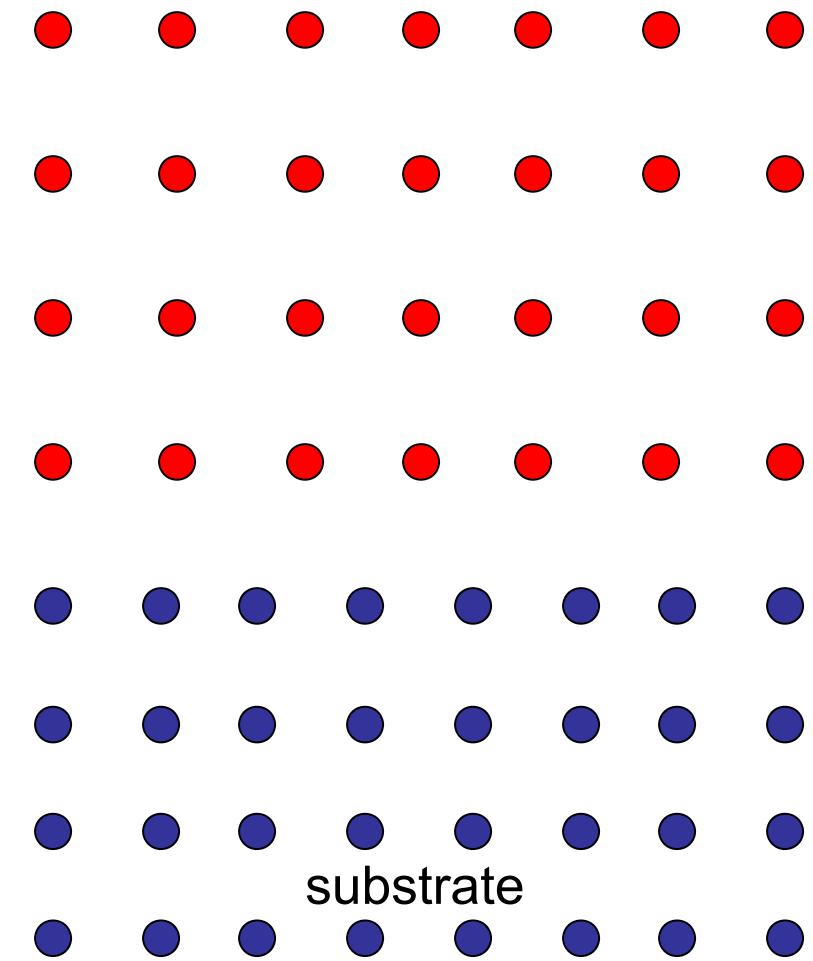
Reciprocal Space

Specular rods overlap



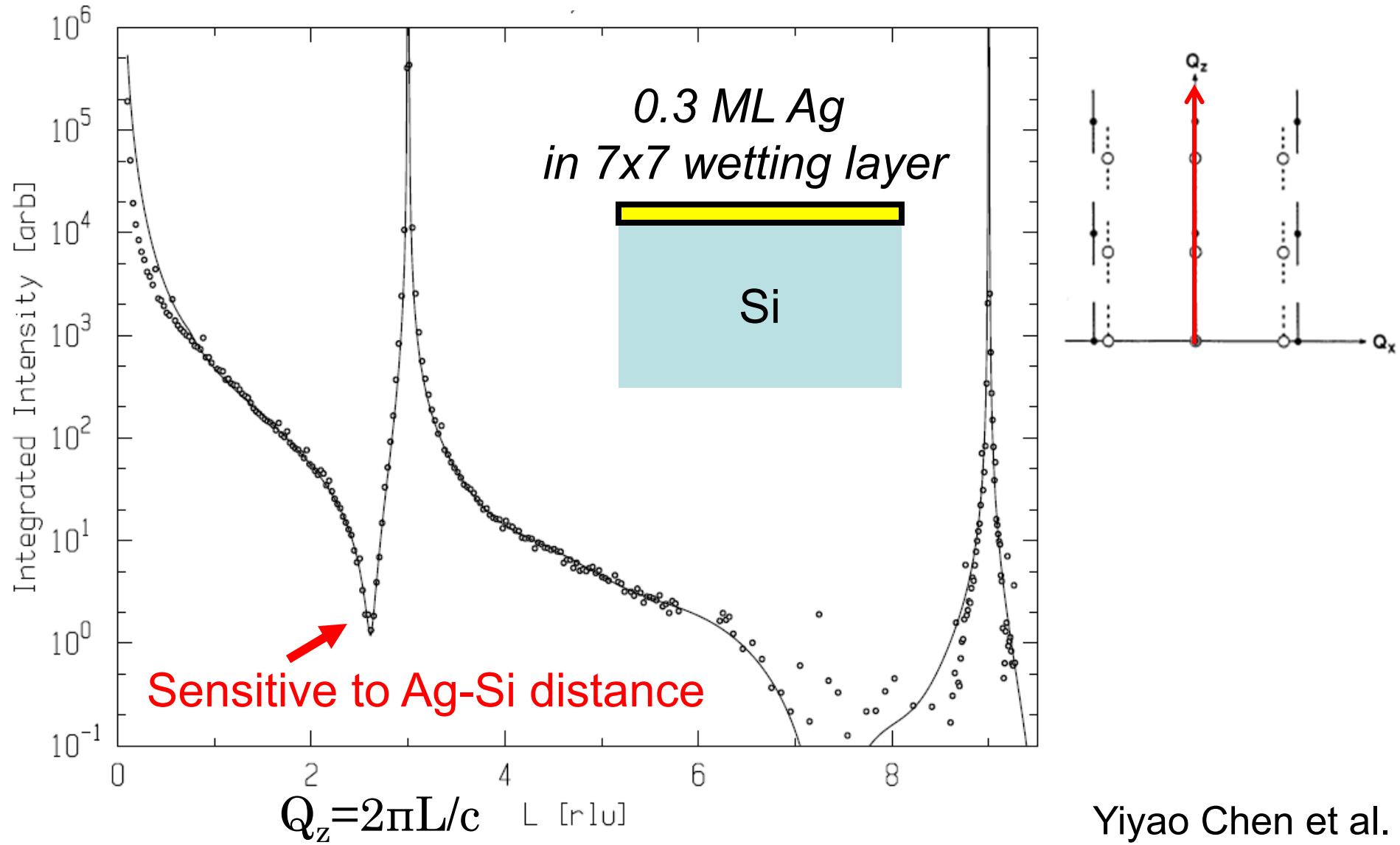
Non-specular rods:
No overlap (incommensurate)

Real Space



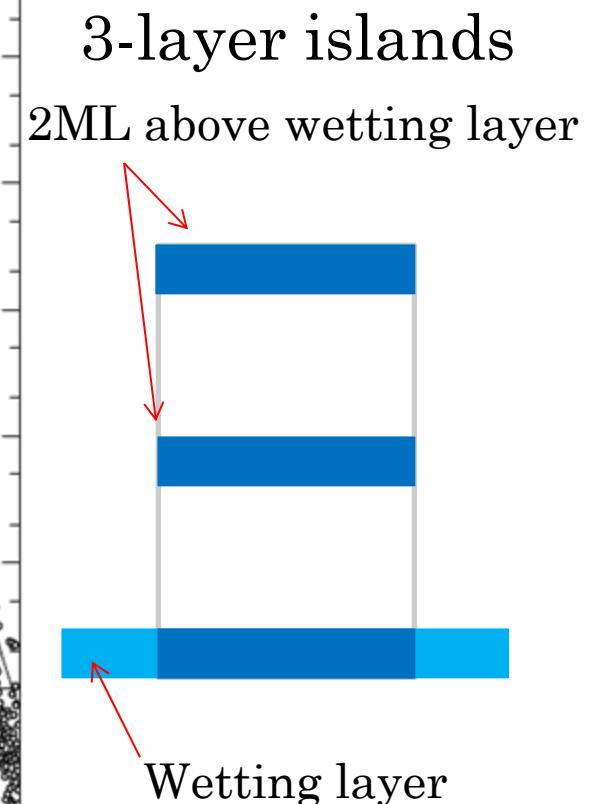
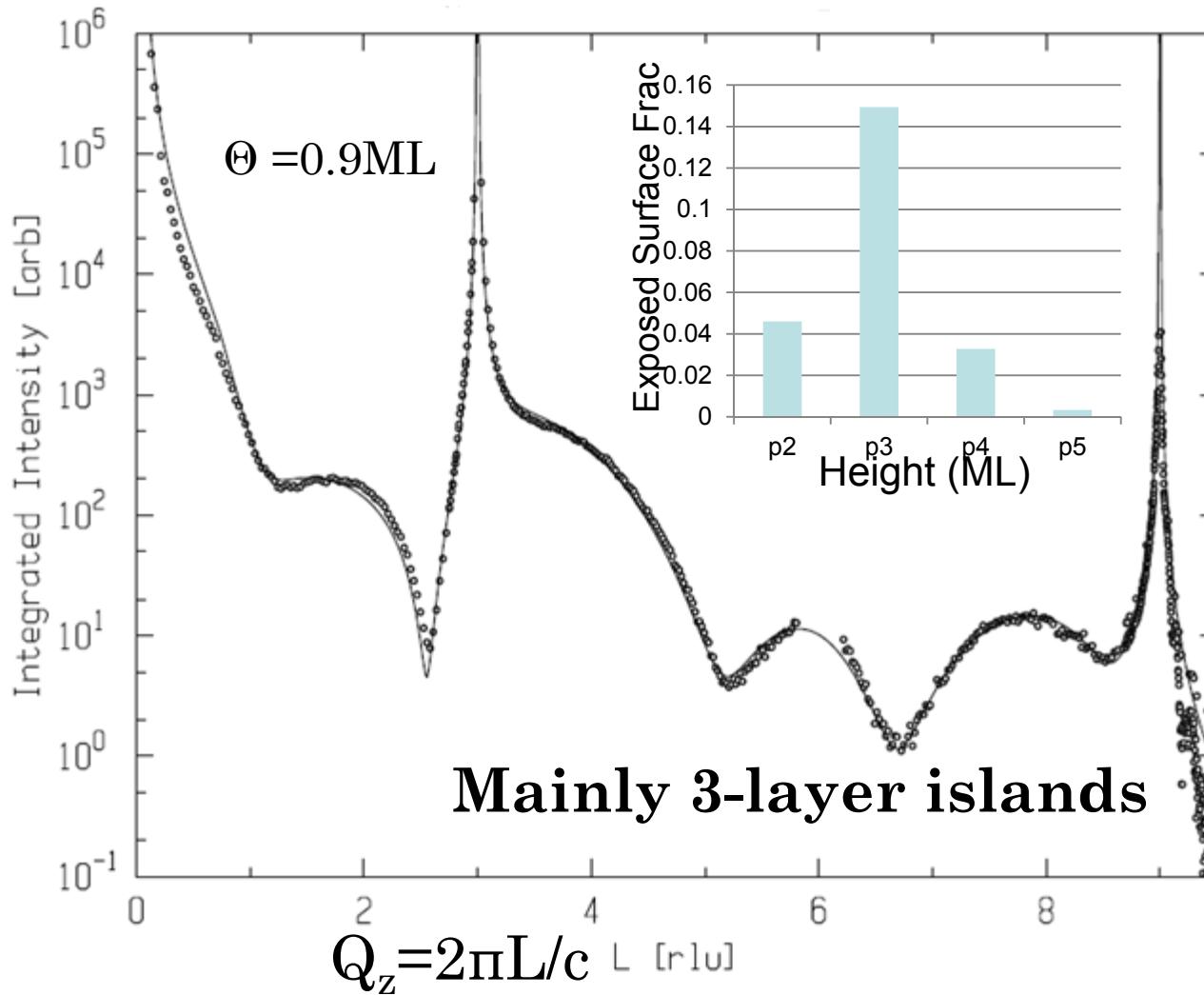
Ag/Si(111)7x7

Specular Reflectivity: 0.3ML Ag/Si(111)7x7



Ag/Si(111)7x7

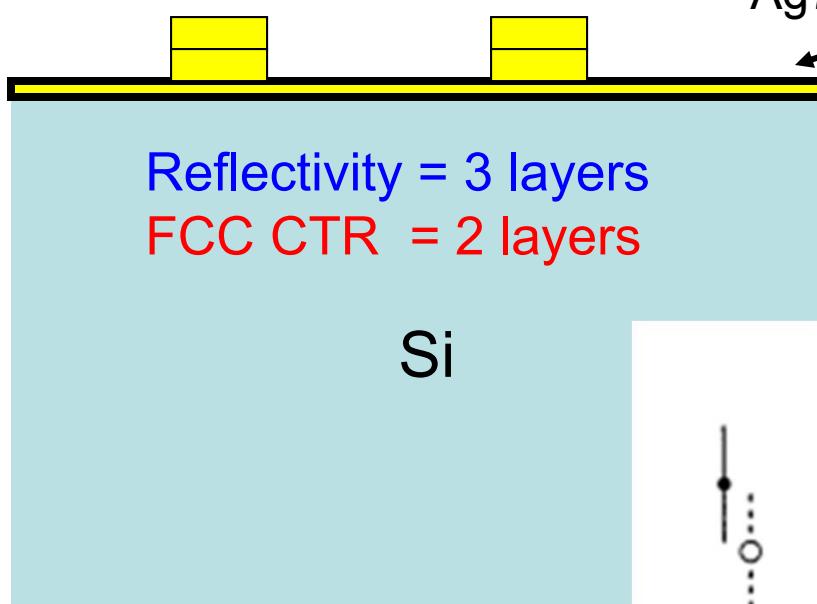
Specular Reflectivity: 0.9ML Ag/Si(111)7x7



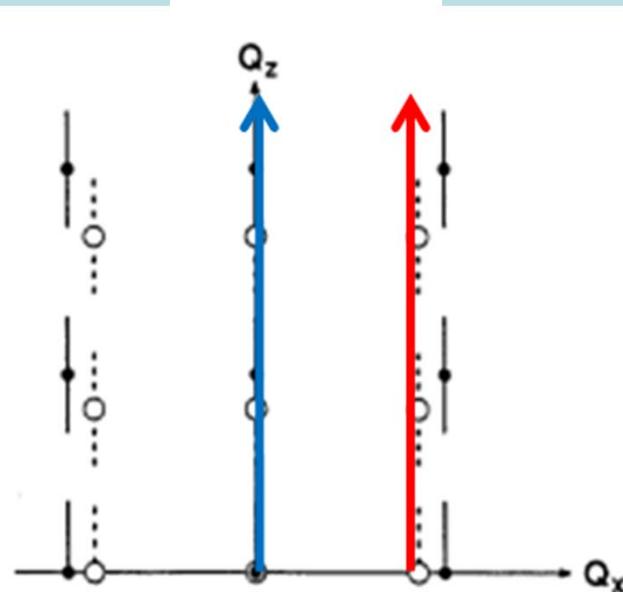
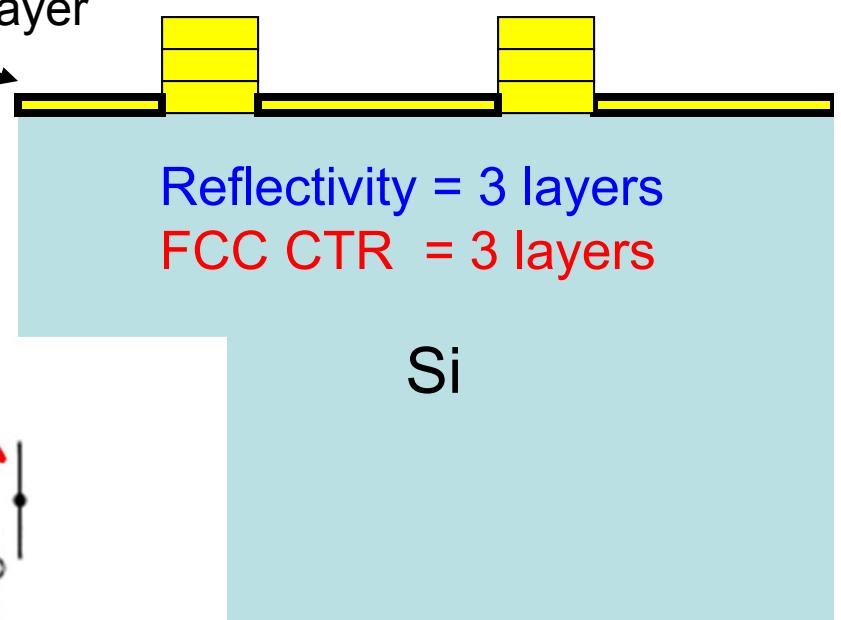
Yiyao Chen et al.

Specular reflectivity cannot easily distinguish between these two cases:

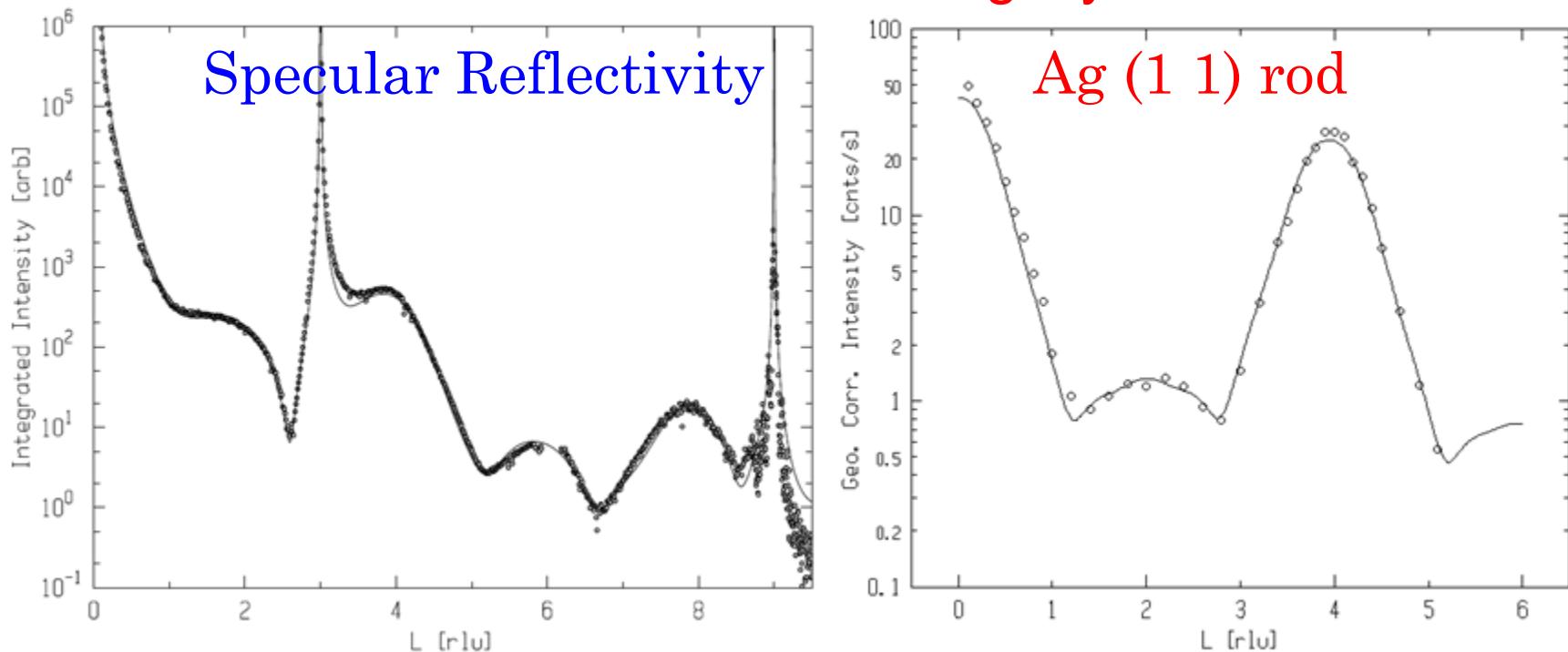
FCC Ag islands on
a Ag 7x7 wetting layer?



FCC Ag islands all the way
to the substrate?

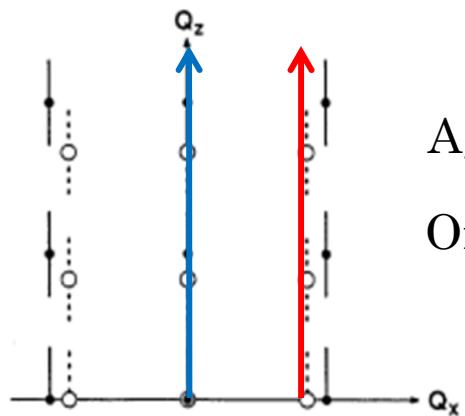


Specular reflectivity and rod give same thickness:
 Island is FCC Ag all the way to the substrate
 Islands *remove* the wetting layer!



↑ Specular Reflectivity
 Probes Si substrate, Ag
 wetting layer and Ag island

↑ Q_z
 Ag truncation rod
 Only probes FCC Ag



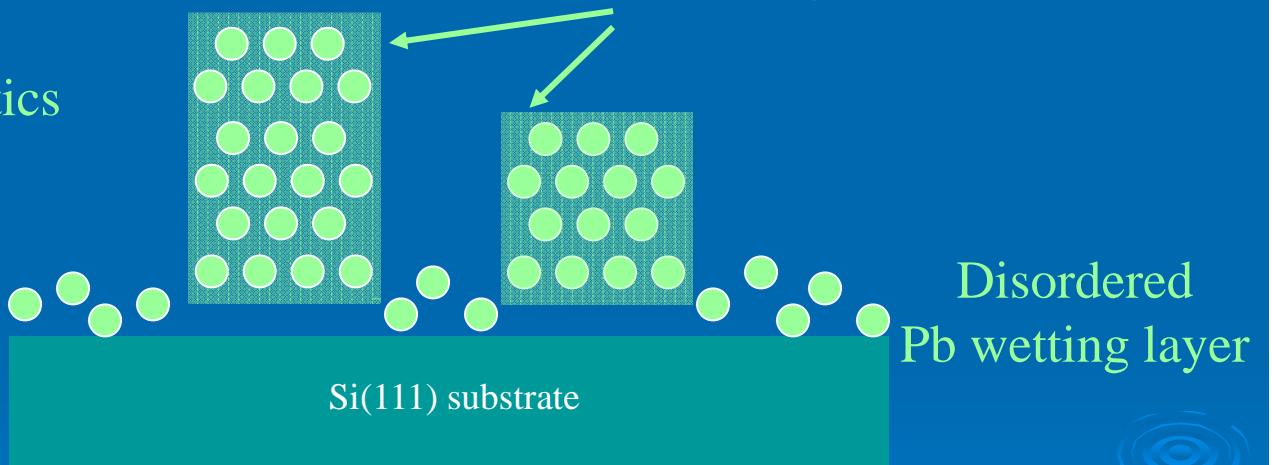
Quantum-Size-Effects: Pb Nanocrystals on Si(111)7x7

Height Selection: “Magic” crystal heights

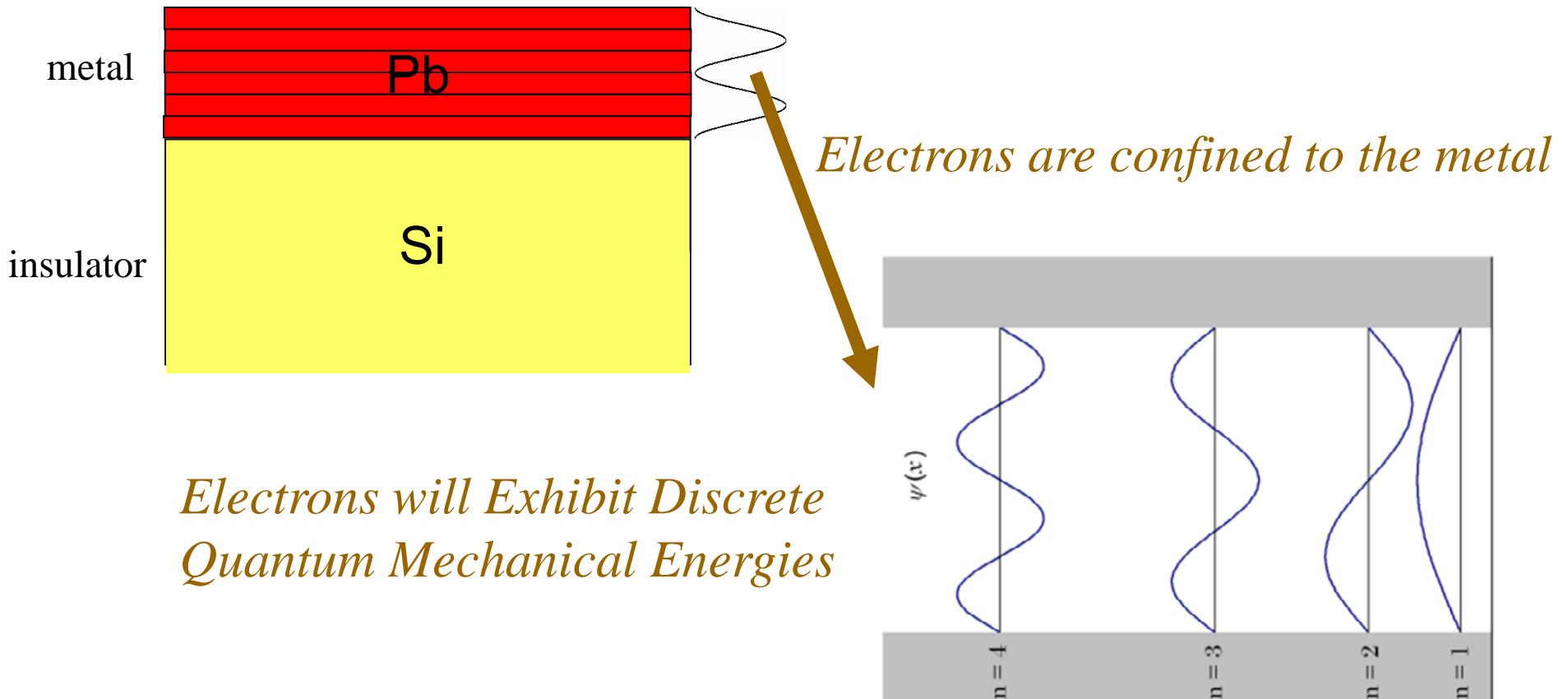
Quantum Mechanics Influences Nanocrystal Growth

Discoveries:

- anomalously (10^4) fast kinetics
- Non-classical coarsening
- Unusual behavior:
fast growth \Rightarrow most stable structures

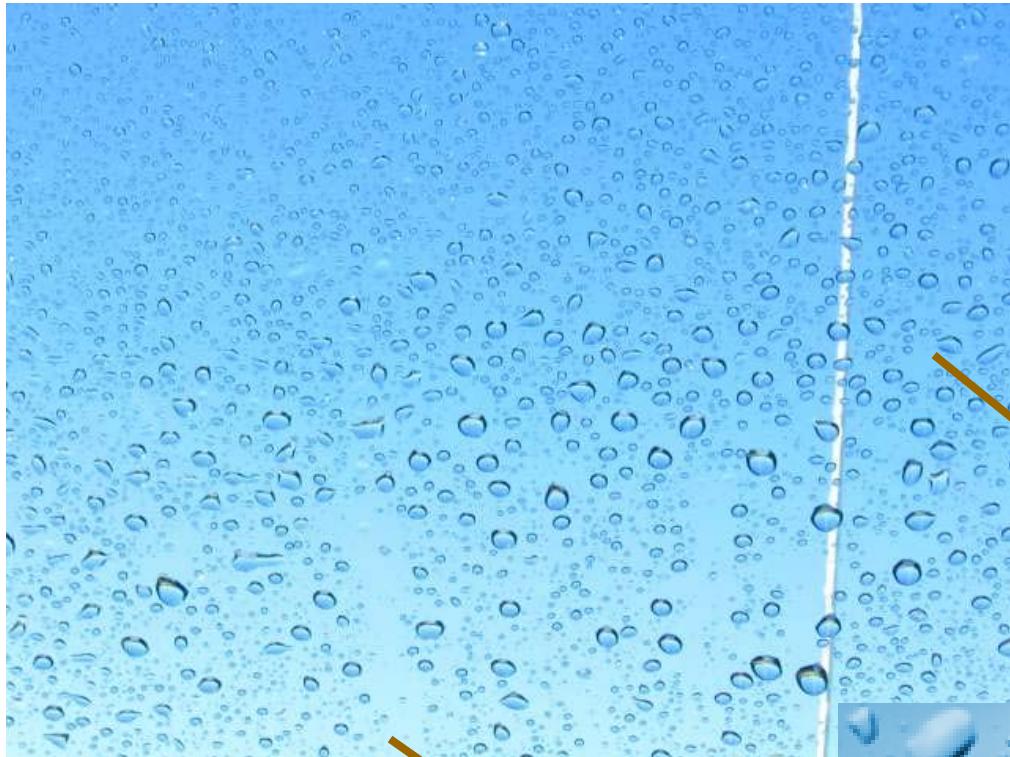


Electrons in a “box”



F. K. Schulte, Surf. Sci. **55**, 427 (1976)
P. J. Feibelman, PRB **27**, 1991 (1983)

physchem.ox.ac.uk

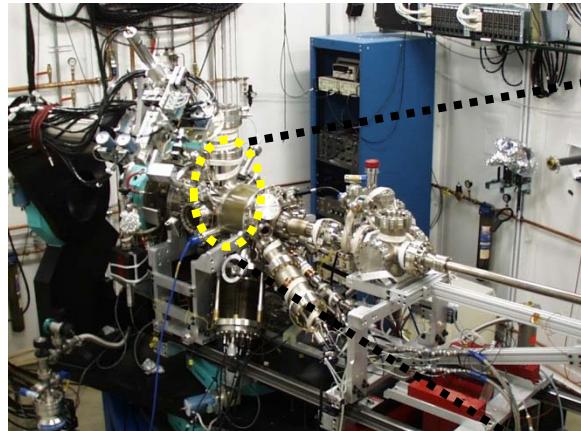


Coarsening



Rain Drops On Your Winshield

Kinetics: In Situ Surface X-ray Scattering



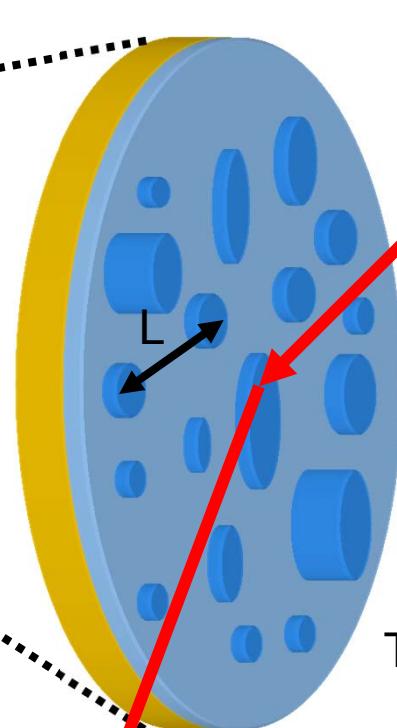
Surface Chamber
Advanced Photon Source

Mean island separation, $\langle L \rangle$:

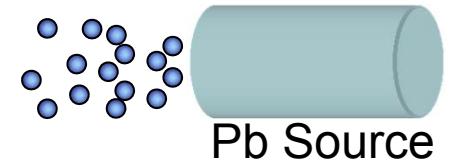
$$\text{Experiment} = \frac{2\pi}{\langle L \rangle}$$

- Deposit Pb ($1.2 \rightarrow 2 \text{ ml}$) at 208K
- Measure the island density

$$(\text{flux off}) \frac{n_{\text{off}}}{\langle L \rangle^2}$$

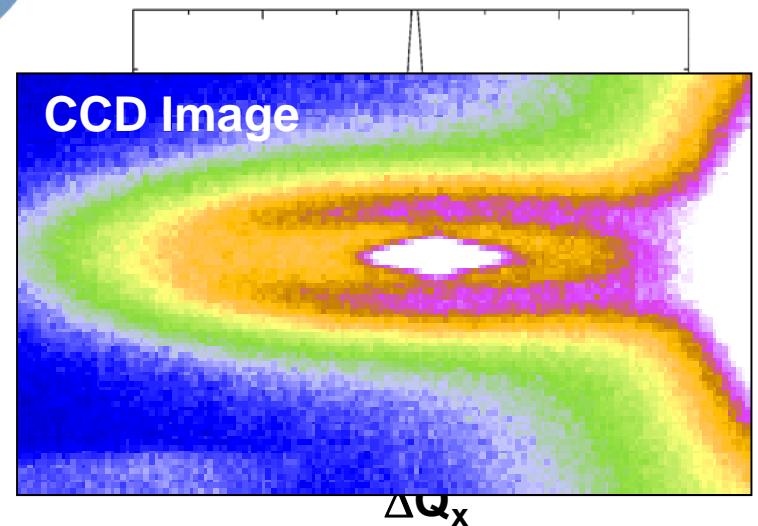


X-Rays Incident



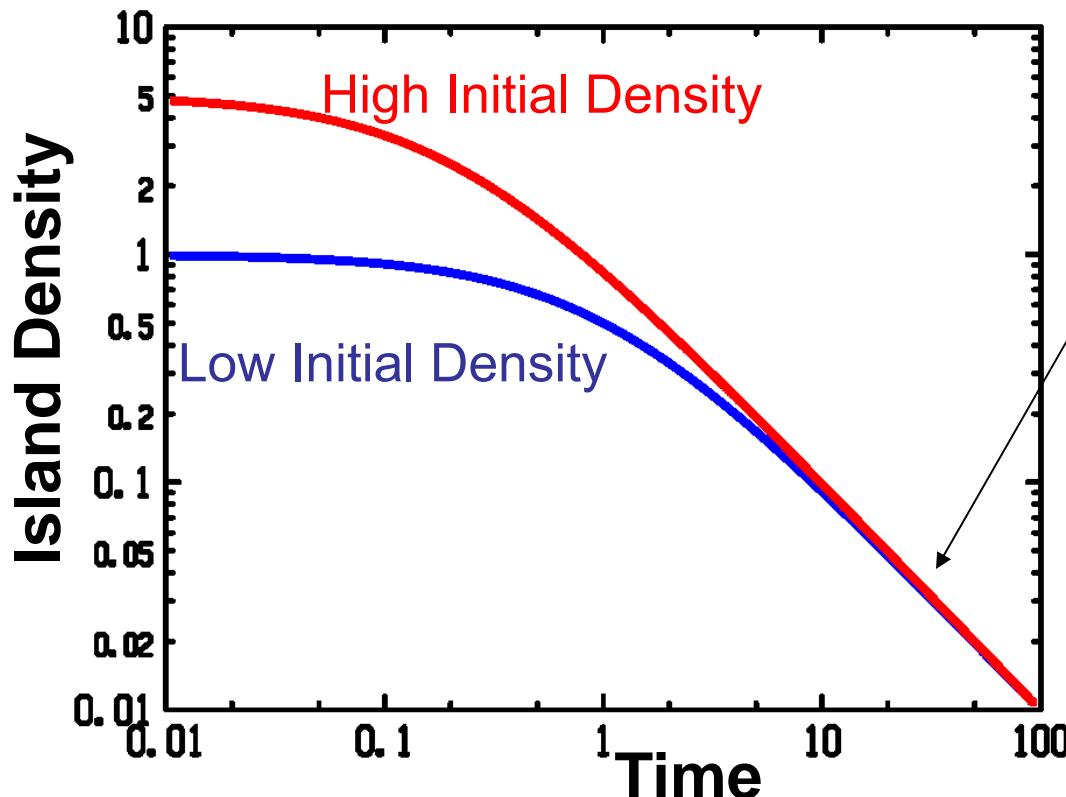
Pb Source

Transverse Scan: In-plane Info.



Classical Coarsening: Ostwald Ripening

$$n(t) = n_0 \left(1 + \frac{t}{\tau}\right)^{-\beta} \quad \beta = \frac{2}{(m+2)}, m = 0, 1, 2$$



Long time: independent
of initial conditions

$$n(t) \Rightarrow (n_0 \tau^\beta) t^{-\beta}$$

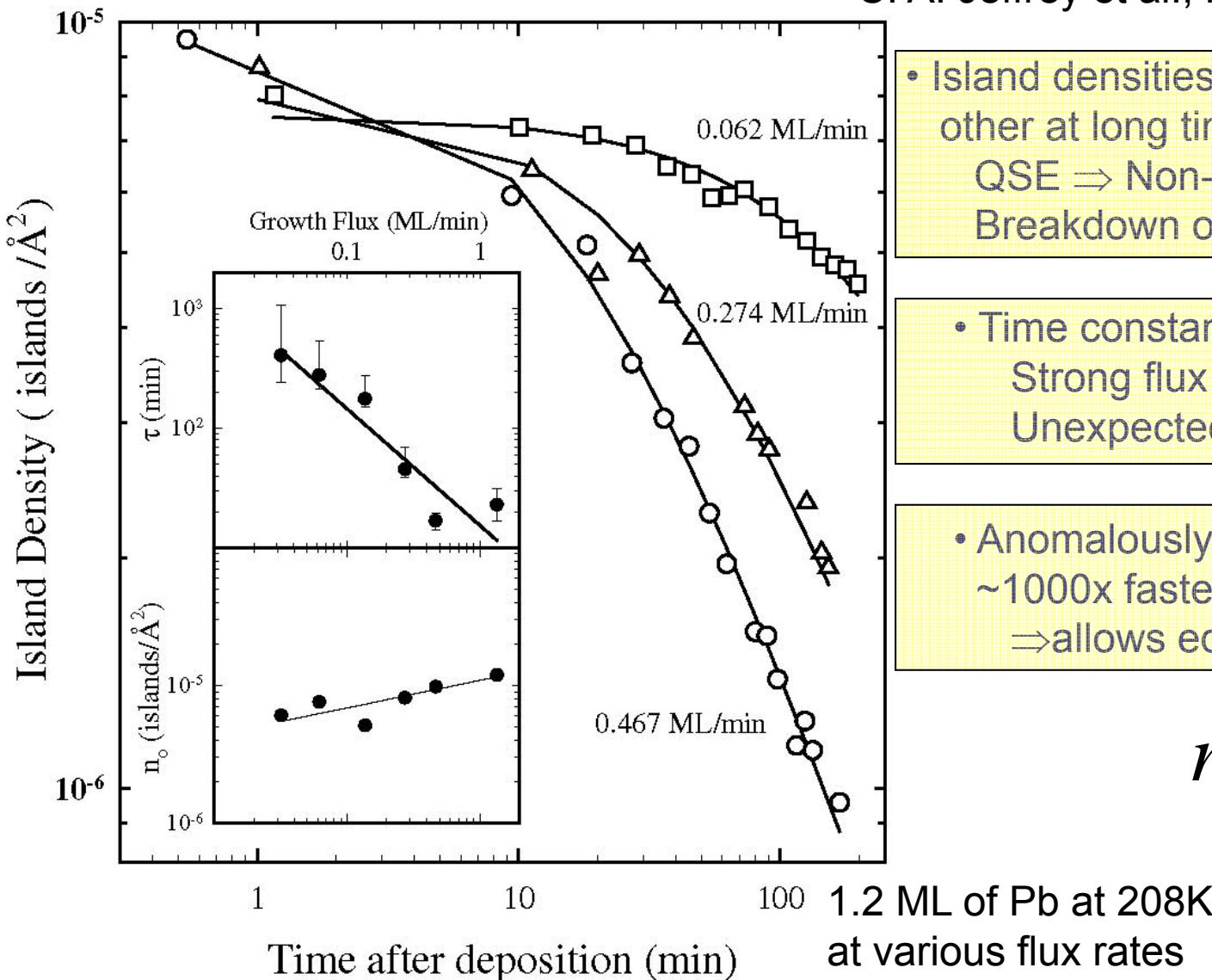
$$\tau \propto n_0^{-\frac{1}{\beta}}$$

Relaxation time depends
only on the initial density

Pb Nanocrystal Coarsening

...does **not** conform to the classical picture!

C. A. Jeffrey et al., PRL 96, 106105 (2006)



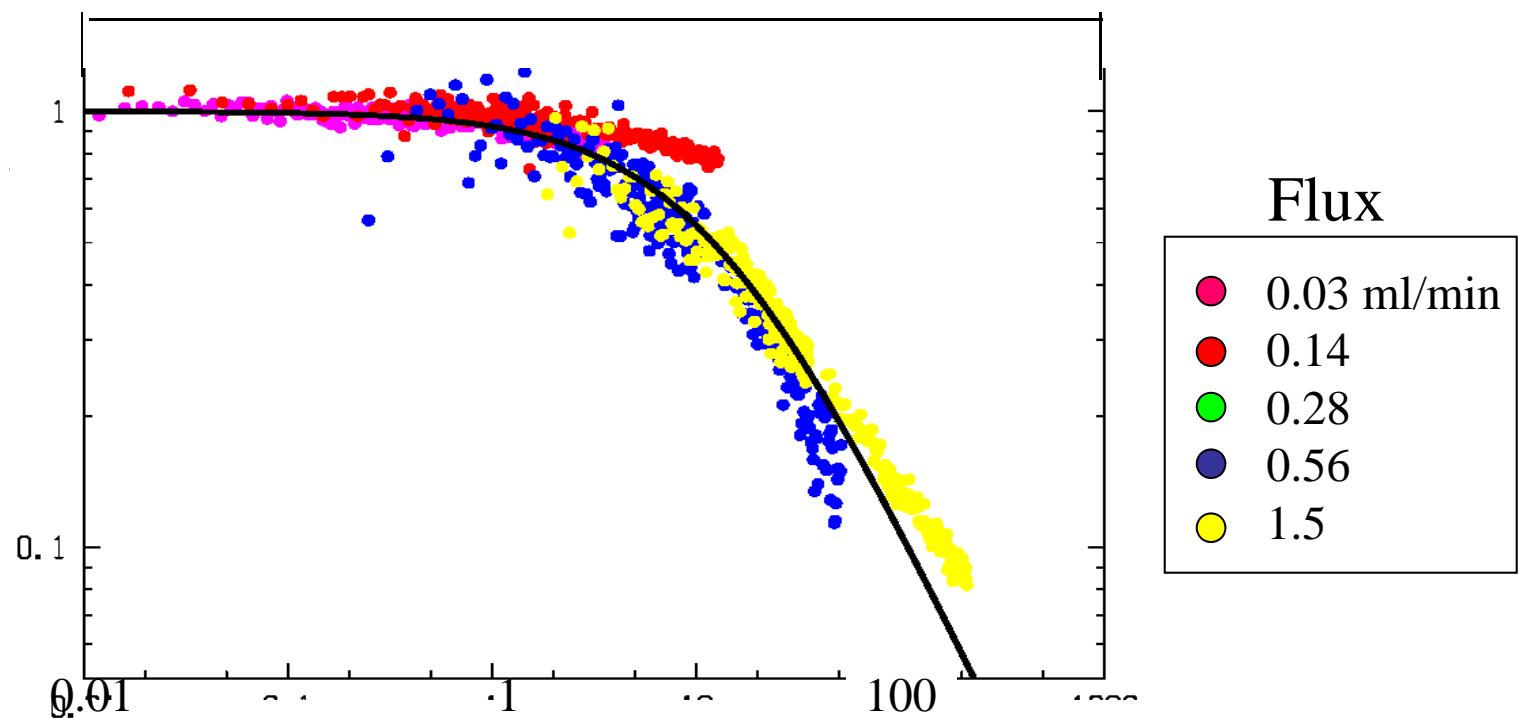
- Island densities do not approach each other at long times:
QSE \Rightarrow Non-Oswald Breakdown of Classical Coarsening

- Time constant $\tau \sim 1/\text{Flux}$
Strong flux dependence!
Unexpected!

- Anomalously fast relaxation
~1000x faster than expected!
 \Rightarrow allows equilibrium!

$$n(t) = \frac{n_0}{\left(1 + \frac{t}{\tau}\right)}$$

Reciprocal Space is Superb for Obtaining Good Statistics of Distributions



$$\text{Equivalent ML Time} = t^*F$$

Summary

- Materials research problems require information on a broad range of length scales, from atomic to mesoscale
- Scattering from surfaces involves many different types of measurements:
 - Reflectivity, Rods, Grazing Incidence Diffraction, Diffuse Scattering
- Unique ability of x-rays: surface and subsurface structure simultaneously