PROPOSAL FOR EXPERIMENTS ON WAVE FUNCTION LOCALISATION

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Abstract

Well-known manifestations of quantum effects in accelerators are related to the quantum fluctuations of synchrotron radiation and the spin polarisation. There are more possibilities to see quantum mechanics signatures in accelerators. It is shown that Bell's inequalities for nonidentical particles, as well as the wave function localisation due to measurement, could be tested with conventional accelerator components and techniques.

1 INTRODUCTION

This work was motivated by papers of Vinokurov and his colleagues [1]. They presented studies of one electron trajectories in a storage ring in presence of strong synchrotron radiation, and confirmed the randomness of its motion. The experiments showed that the electron wave function didn't spread out over the ring but, instead, its the localisation is fairly good and "the possible cause of the localisation is the interaction with the radiation field, which acts like a continuos measurement". Thus, the nontrivial question is how and when the localisation occurs and is it related to the measurement (or equivalent) process. In even simpler words, when the electron radiate, when does its entanglement with the photon break?

The quantum mechanical behaviour of particles could be perfectly demonstrated by interference pattern, appearing when particles pass through a double-slit and detected at the screen behind it. For two particles, having correlated spins (or other variables), there exist another exceptional possibility to measure, if the particle wave functions are coupled (entangled) to each other. In 70^{th} John Bell [2] presented a scheme for polarisation measurement for two particles, which gives different results for some correlation function (S) of particle spins, if measured on various axes, in case when two particle spins are entangled or independent. The function S, as well as three related examples, is described in appendix. The paper deals with the simple set-up to check the entanglement of the photons and to check if the process of conversion of polarised photon into a polarised electron breaks the entanglement.

2 IS SUPERLUMINAL COMMUNICATION POSSIBLE?

In order to show what type of experiments could be done with entangled photons, we present an experimental scheme that deals with possibility to transmit signals faster than the speed of light.

Let's take the source of entangled photons, moving in opposite directions, having equal (entangled or coupled to each other) polarisation. For every photon wave function consists of combination of parallel and perpendicular polarisation, with the wave function ψ

$$y \propto \left|1_{\perp},2_{\perp}\right\rangle + \left|1_{\parallel},2_{\parallel}\right\rangle, \quad (1)$$

where symbols ⊥ and || mean perpendicular and parallel polarisation with respect to some axis.[1] One can see that this wave function can not be factorised and the photons in this state are called entangled photons. The remarkable feature of this state is that if one photon polarisation is measured to be equal to some value (e.g. angle in radians with respect to some axis), the opposite photon instantly has the same polarisation.

Figure 1 shows the simplest experimental set-up with the future experimental outcome to be a puzzle from theoretical point of view. The scheme consists of two parts.

The right part is just polarisation measurement scheme, which consists of polarisation splitter cube and two Photo Multipliers (PMT's) to count the photons, and the switch, which either deflects the beam from the cube or directs the beam into it. We assume that the deflected beam is travelling without measurement. But if the photon passes through the cube, its polarisation is measured with 100% efficiency.

The left part is an interferometer (the principal scheme is taken from 3). The photon in state (1), travelling left, is splitted in the cube with parallel polarisation transmitted through it and perpendicular reflected down. We assume that the reflection in half-transparent mirror shifts the phase of the light to quarter of oscillation, the regular mirrors and polarisation splitter for the perpendicular polarisation change the oscillation phase to half, and each direct passing of light through them leaves the phase unchanged. In addition, we assume that the phase shifter below the cube shifts the oscillation phase to -π/2. Parallel polarisation is transformed into perpendicular after the upper left mirror

[1] Therefore the polarisation of every photon, if measured separately, is equal for any axis

in order to have interference pattern for both photon paths. It easy to calculate the phase difference for both photon paths and find that, if the photon polarisation is not measured at the right shoulder of the set-up, the Photo Multiplier A (PMT A in the scheme) has zero phase difference between two photon paths[2]. Thus it has nonzero signal, while the PMT B has π phase advance and no signal.

The described above scheme gives interference if the photon is not measured in the right side PMT's. If it is not deflected by a fast switch and its polarisation is measured, it automatically travels through one path (which correlates with the results of the measurements on the right), and both PMT A and PMT B have statistically equal signals. Thus if the collapse of two photon wave function is instant, we can transmit signals faster than the speed of light at the statistical level.

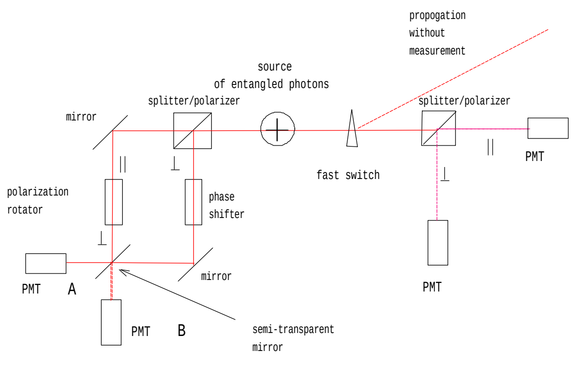

Figure 1 Experimental set-up with interference pattern to disappear at the left if the polarization measured at the right.

3 DOES THE CATHODE BREAK THE ENTANGLEMENT?

Let's take the source of entangled photons similar to the described in Section 1. Figure 2 shows proposed experimental set-up to measure if the absorption of the photon and related to it emission of the electron breaks the entanglement with another photon. If taken without the cathode, the set-up resembles one use by Alain Aspect and his colleagues [4]. The entangled photons after splitting in polarimeter cubes are measured by photomultipliers (PMT) at the left and converted to polarised electrons at the right, which are measured by Mott polarimeter Microchannel Plates (MCPs). Cube I directs different polarisations into two photon channels. These photons are transformed into circular photons and produce electrons with longitudinal polarisations (forward or backward, depending on the polarisation of the incident photons). The spin rotator converts the longitudinal polarisation into a transverse one. Therefore measured electron spin "up" corresponds to the photon with the parallel polarisation, and spins "down" - to the perpendicular polarisation.

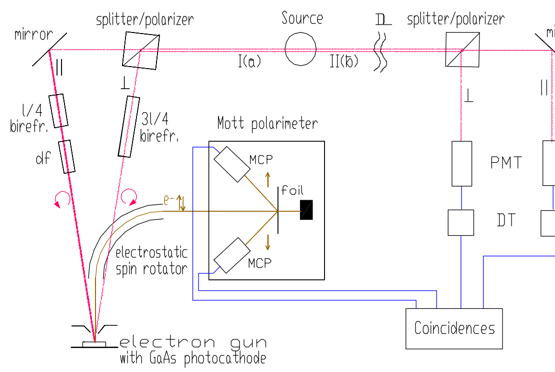

Figure 2 Proposed experimental setup. Two polarimeters I, in orientation a, and II, in orientation b, split the first photon to produce polarized electrons (at the left part) and to direct the second one to two photomultipliers (at the right side). Each polarimeter is rotatable around the axis of the beam.

4 POSSIBLE SCHEME TO CHECK THE EQUIVALENCE BETWEEN RADIATION AND MEASUREMENT

Finally, we come to the questions of paper [1]. The studies, presented in it, show one-electron trajectory in a storage ring in presence of strong synchrotron radiation. The experiments showed that the electron wave function didn't spread out over the ring but, instead, its the localisation is fairly good and "the possible cause of the localisation is the interaction with the radiation field, which acts like a continuos measurement". Thus, the nontrivial question is how and when the localisation occurs and is it related to the measurement (or equivalent) process. Or, in reformulated form, when the electron radiate, when does its entanglement with the photon break?

It turns out to be that there exists already optical scheme for accelerators (namely, ATF in KEK, Japan) to measure sizes of the electron beam with the interferogram [5]. Figure 3 shows the rough scheme of the experiment.

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[2] The upper left path has reflection-related phase shift in PMT A equal to π (due to the upper left mirror) + π/2 (due to the half-transparent mirror) = 3π/2. The lower path has the phase shift π(due to the cube)-π/2 (due to the phase shifter)+ π(due to the mirror)=3π/2, therefore the phase shifts are equal. Similar calculation gives π phase difference between two paths and, therefore, no signal in PMT B.

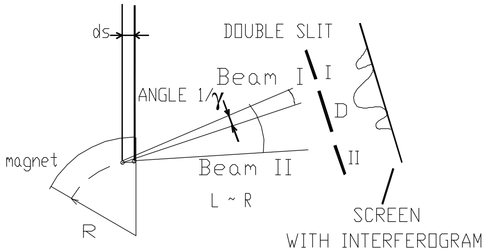

Figure 3 Principal scheme of the SR experiment

The circles at the Figure 3 show two consecutive positions of an electron. It radiates synchrotron radiation into angle $1/\gamma$. The double slit is placed perpendicularly to the Synchrotron Radiation (SR) light. Light Beam I and Light Beam II represent two different situations. Beam I corresponds to the situation when the SR light breaks away from the electron at the short distance (the act equivalent to measurement happened). The width at the figure is $1/\gamma$ as if the act of radiation occurred instantly. If the width of the beam is less than the distance D between the slits, the interference pattern is absent. Beam II represents the classical situation when the radiation field is running along with the electron. In this case both slits see approximately the same intensity (at the same time, because the electron is ultrarelativistic and its trajectory is perpendicular to the slits) and produce the interferogram.

Paper [4] has presented the results for the visibility versus the distance D between slits. Roughly, the interferogram disappears when this distance is about 30 mm. The authors relate this fact to the horizontal beam size, which was calculated to be equal to 39 μm.

These number impose some limitations on the length when the photon become independent of the electron. First of all, if the process of radiation happens instantly, the size of the light spot would be about $L/\gamma \approx 2$mm (for $R \approx 6$m, $L \approx 7$m, $\gamma \approx 3000$). Since the interferogram disappear for much larger distances (30 mm), this possibility is ruled out. If we assume that the beam actual size is much smaller than 39 μm, and the interference pattern disappear because of the photon becomes independent and its entanglement with the electron breaks, the distance of this ("measurement-like") wave function localisation process is about 30 mm. This could be regarded as the lower limit for the "measurement" distance for the ATF set of parameters.

We think that it is of a great interest to combine both [1] and [4] experiments, to measure the distance of the photon localisation process, the "size" of one electron, etc.

CONCLUSION

Three schemes to check when the quantum measurement happens are presented. All look realisable with modern accelerators or their components.

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APPENDIX

The Bell's inequalities apply to various types of problems. We follow Ref.[6] to present general formulation for the inequalities, and then, three examples for photons in various states. We deal with correlation for variables of two particles, for example, with equal (but not necessarily definite) polarisation, for the first photon to be polarised on axis with the angle a and for another photon, to have polarisation with the angle b. The statistical correlation function is denoted as $P(a,b)$. The second conceptual definition is one for the hidden variables. According to the Einstein locality principle, the certain event (e.g., one of the photons polarisation is measured to be equal to a) occurs with probability, calculated in simple way as a sum of possible events (or paths of integration) λ, with the density $\rho(\lambda)$. The dichotomic variable, which takes the values only ± 1(+1 when the variable, for example polarisation angle of the photon, equal to a, and -1 when the polarisation is perpendicular to a) in the case of event l is denoted as $A(a, l)$, and for the second particle - $B(b, l)$. The correlation function for two particles to have a and b variables, is just multiplication of $A(a,l)$ and $B(b,l)$ (as for the independent events) with the integration of the result over all the l with the density $\rho(\lambda)$. Let's a, b, c … be the adjustable apparatus parameters (e.g., axes of polarimeters). Let's look at the following inequality:

$$| P(a,b) - P(a,c) | \leq \int_{\Gamma} | A(a,l)B(b,l) - A(a,l)B(c,l) | r(l) dl \; . \quad (1a)$$

The inequality is just a consequence of the fact that the sum of modulus is larger than modulus of the sum. Since $|A(a,l)|=1$, the R.H.S. of the previous equation could be transformed in the following way:

$$| P(a,b) - P(a,c) | \leq \int_{\Gamma} | B(b, l) - B(c, l) | r(l) d l \quad \text{(2a)}$$

Similar mathematics could be used to show that

$$| P(d,b) + P(d,c) | \leq \int_{\Gamma} | B(b, l) + B(c, l) | r(l) d l \,, \quad \text{(3a)}$$

Because $|B(b,l)|= |B(b,l)|=1$, *we have:*

$$| B(b,l) - B(c,l) | + | B(b,l) + B(c,l) | = 2. \quad \text{(3a)}$$

Using this and adding (2a) and (3a), we have: *|P(a,b)-P(a,c)|+| P(d,b)+P(d,c), |£ 2.* The consequence of this inequality is also the following inequality:

-2 £ P(a,b)-P(a,c)+P(d,b)+P(d,c) £ 2. (4a)

It will be referred to as Bell's inequality. The surprising result that Bell obtained was that the quantum mechanical calculations violate the inequality.

To show the example, we denote *P(a,b)-P(a,c)+P(d,b)+P(d,c)* as S with respective subscripts (and we call S a Bell's function).

Let's examine three cases to see the difference between quantum mechanical predictions and the above inequalities. Let's take angles *a,b,c,d* equal to *a=q, b=z+q, c=2z+q, d=3z+q* and scan the inequalities versus the parameter *z*. Three cases for testing are chosen to be:

1) S_1- calculations for the zero angle both photon polarisation (*q=0*);

2) S_2 - calculations for equal but decoupled photon polarisation (uniform distribution over *q*);

3) Q - quantum mechanical prediction for the photon state 1/sqrt(2)(parallel + perpendicular).

P(x,y)=w(x+,y+)- w(x+,y-)- w(x-,y+)+w(x-,y-), (5a)

for any axes *x,y*, where *w* functions stands for probability, + and - signs stand for the case when photon has parallel and perpendicular polarisation on the chosen axis, respectively. For the first case, when the photons polarisation angle is equal to zero, for arbitrary axes *x,y* we have:

$$\begin{aligned} w(x+, y+) &= \cos(x)^2 \cos(y)^2, \\ w(x+, y-) &= \cos(x)^2 \sin(y)^2, \\ w(x-, y+) &= \sin(x)^2 \cos(y)^2, \\ w(x-, y-) &= \sin(x)^2 \sin(y)^2. \end{aligned} \quad \text{(6a)}$$

Using (5a) one can get S_1 (see formula (4a)) for the first case. For the second case for arbitrary axes *x,y* we have:

$$\begin{aligned} w(x+, y+) &= \int_0^{2p} \cos(x+q)^2 \cos(y+q)^2 \frac{dq}{2p}, \\ w(x+, y-) &= \int_0^{2p} \cos(x+q)^2 \sin(y+q)^2 \frac{dq}{2p}, \\ w(x-, y+) &= \int_0^{2p} \sin(x+q)^2 \cos(y+q)^2 \frac{dq}{2p}, \\ w(x-, y-) &= \int_0^{2p} \sin(x+q)^2 \sin(y+q)^2 \frac{dq}{2p}. \end{aligned} \quad \text{(7a)}$$

The integration over *q* is just consequence of the uniform distribution of the initial state over the angle. One can get S_2 easily from (5a) and (8a). The last one is quantum mechanical case, when the expression under the integrals are not just multiplication of independent probabilities as functions of *x* and *y*. The polarisation of the second photon now depends on what axis the first photon was measured, therefore the second photon polarisation depends on the difference of angles *x* and *y*. The probabilities are:

$$\begin{aligned} w(x+, y+) &= \int_0^{2p} \cos(x+q)^2 \cos(x-y)^2 \frac{dq}{2p}, \\ w(x+, y-) &= \int_0^{2p} \cos(x+q)^2 \sin(x-y)^2 \frac{dq}{2p}, \\ w(x-, y+) &= \int_0^{2p} \sin(x+q)^2 \cos(x-y)^2 \frac{dq}{2p}, \\ w(x-, y-) &= \int_0^{2p} \sin(x+q)^2 \sin(x-y)^2 \frac{dq}{2p} \end{aligned} \quad \text{(8a)}$$

Again, using (5a) and (7a) one can get Q. Figure 1a shows all three functions S_1 (red solid line), S_2 (blue dotted line), and Q (green dashed line) as functions of the relative angle *z* between axes *a, b, c, d*. One can see that the quantum mechanics prediction for two entangled photons (Q as a function of the relative angle z) violates Bells inequalities. The strongest violation occurs for 22.5 angle (and combinations of this angle with the integer numbers of ninety degree angle), where Q=2√2. Another method to calculate the quantum mechanical case, more formal and scientific, could be found, for example, in [7].

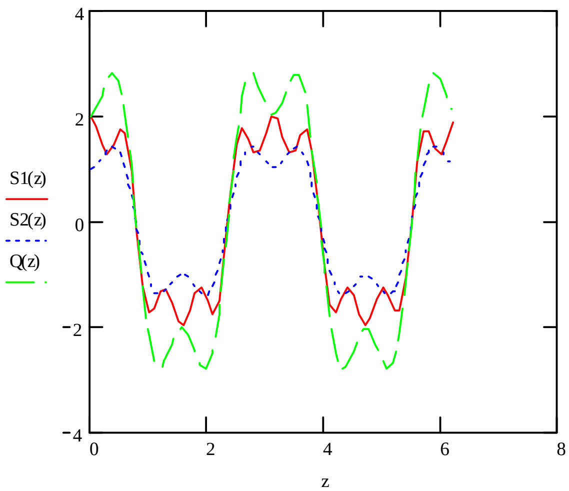

Figure 1a Bell's function versus the relative angle of the polarimeter axes. The red and the blue lines represent independent photons, the green one - two entangled photons.

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