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Note on the observation of quadrupole oscillations in the PSR beam.

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Abstract

This is a note on the observation of quadrupole oscillations in PSR beam using signals from strip line BPM. Sensitivity of BPM to quadrupole oscillations is calculated and results of signal processing are presented.

I. Introduction

Besides of beam centroid movement beam size or shape can change periodically along the beam trajectory. It is usually treated as oscillations of higher moments of charge density distribution. Quadrupole oscillation is the simplest mode after dipole oscillation of beam centroid. It was shown [1] that quadrupole oscillations or so called breathing modes can lead to halo growth and increase beam losses. This kind of instability can develop in PSR beam due to space charge as was predicted by computer simulation [2]. To check this prediction special experiment was carried out where PSR tunes were adjusted to unstable values (ν_x=3.17, ν_y=2.14) predicted by the simulation code [2].

In this work we analyze signals from top, bottom, left and right strip lines digitized separately to identify and measure quadrupole oscillations. Sensitivity of strip line PSR BPM to beam size changes is calculated to obtain quantitative results. Longitudinal quadrupole oscillations are observed while there is no evidence of transverse oscillations.

II. BPM sensitivity to the moments of transverse charge distribution.

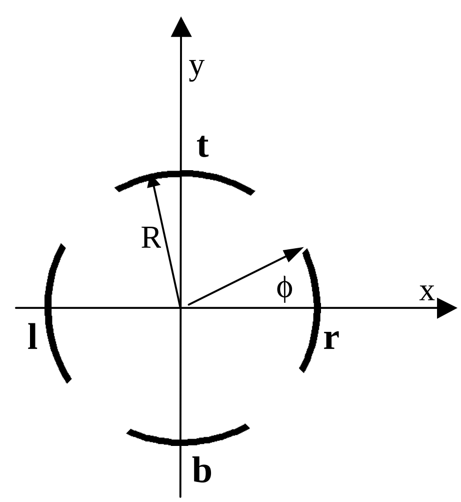

Figure 1. A simplified model of a 4-lobe strip line BPM.

Lets consider 4 lobe BPM schematically shown in fig.1. The voltage induced on the n-th electrode can be calculated from beam current transversal distribution ρ(x,y) and electrode sensitivity function $u_{n0}(x,y)$:

$$u_n = \iint_{aperture} u_{n0}(x,y)\, r(x,y)\,dxdy\ , \qquad (1)$$

u_{n0} can be developed to 2D power series about center of the BPM :

$$u_{n0} = \sum_{i,j=0}^{\infty} a_{ij} x^i y^j\ .$$

Substituting it to (1) we have:

$$u_n = \iint_{aperture} \sum_{i,j=0}^{\infty} a_{ij} x^i y^j\, r(x,y)\,dxdy = \sum_{i,j=0}^{\infty} a_{ij} \iint_{aperture} x^i y^j\, r(x,y)\,dxdy = \sum_{i,j=0}^{\infty} a_{ij} m_{ij}\ ,$$

where m_{ij} are 2D moments of the beam current distribution. Let u_t, u_b, u_r, u_l be a voltage induced on top, bottom, right and left lobes respectively. BPM symmetry about its axis imposes some restrictions on the sensitivity functions: ut and u_b have to be even functions of x; u_r, u_l have to be even functions of y; $u_b(x,y)= u_t(x, -y)$; $u_r(x,y)= u_l(-x,y)$; $u_r(x,y)= u_t(y, x)$; $u_l(x,y)= u_b(y, x)$. Then these functions can be written as:

$$\begin{aligned}
u_{t0} &= a_{00} + a_{01}y + a_{20}x^2 + a_{02}y^2 + a_{21}x^2y + a_{03}y^3 + \ldots \\
u_{b0} &= a_{00} - a_{01}y + a_{20}x^2 + a_{02}y^2 - a_{21}x^2y - a_{03}y^3 + \ldots \\
u_{r0} &= a_{00} + a_{01}x + a_{20}y^2 + a_{02}x^2 + a_{21}y^2x + a_{03}x^3 + \ldots \\
u_{l0} &= a_{00} - a_{01}x + a_{20}y^2 + a_{02}x^2 - a_{21}y^2x - a_{03}x^3 + \ldots
\end{aligned}$$

Some useful combinations of the signals are:

1. Sum signal: $u_{\Sigma 0} = u_{t0} + u_{b0} + u_{r0} + u_{l0} = 4a_{00} + 2(a_{20} + a_{02})(x^2 + y^2) + \ldots.$

2. Vertical difference: $u_{vd0} = u_{t0} - u_{b0} = 2a_{01}y + 2a_{21}x^2y + 2a_{03}y^3 + \ldots.$

3. Vertical sum minus horizontal sum:

$$u_{hv0} = u_{t0} + u_{b0} - u_{r0} - u_{l0} = 2a_{20}x^2 + 2a_{02}y^2 - 2a_{20}y^2 - 2a_{02}x^2 + \ldots = 2(a_{02} - a_{20})(x^2 - y^2) + \ldots$$

In the lowest nonzero order u_Σ is proportional to zero order moment of beam distribution i.e. to full current, u_{vd} and u_{hd} proportional to vertical and horizontal dipole moments respectively, u_{hv} is proportional to difference between the second moments of distribution. To eliminate dependence upon beam current normalization on sum signal is usually used. We are interested in the second moments of the beam distribution therefore we will concentrate on 'vertical sum minus horizontal sum' in the following analysis. After truncating series to the second order and normalization we have:

$$u_q = \frac{u_{hv}}{u_\Sigma} = \frac{2(a_{02}-a_{20})(\langle x^2\rangle - \langle y^2\rangle)}{4a_{00}+2(a_{20}+a_{02})(\langle x^2\rangle + \langle y^2\rangle)} \approx \frac{1}{2}\frac{(a_{02}-a_{20})}{a_{00}}(\langle x^2\rangle - \langle y^2\rangle),$$

where < > means averaging over transversal beam current distribution. In general $\langle x^2\rangle = \overline{x}^2 + \overline{x^2} = \overline{x}^2 + s_x^{\ 2}$; $\langle y^2\rangle = \overline{y}^2 + \overline{y^2} = \overline{y}^2 + s_y^{\ 2}$, where $\overline{x}, \overline{y}$ are center of gravity coordinates and s_x, s_y are r.m.s. sizes of the beam. Finally

$$u_q = \frac{1}{2}\frac{(a_{02}-a_{20})}{a_{00}}(\overline{x}^2 - \overline{y}^2) + \frac{1}{2}\frac{(a_{02}-a_{20})}{a_{00}}(s_x^{\ 2} - s_y^{\ 2}).$$

Note that betatron motion of beam centroid gives contribution to u_q as well as variation of r.m.s. size. Therefore one should find beam centroid position using formulas for the dipole moments and use it in calculation of r.m.s. sizes. Assume that beam is centered on a equilibrium orbit that is offset by d_x,d_y from BPM center and r.m.s. sizes of the beam oscillate around an equilibrium value:

$$s_x = s_{x0}(1+\frac{ds_x}{s_{x0}}\sin w_x t), \qquad s_y = s_{y0}(1+\frac{ds_y}{s_{y0}}\sin w_y t),$$

then

$$u_q = \frac{1}{2}\frac{(a_{02}-a_{20})}{a_{00}}(d_x^{\ 2} - d_y^{\ 2}) + \frac{1}{2}\frac{(a_{02}-a_{20})}{a_{00}}(s_{x0}^{\ 2}(1+\frac{ds_x}{s_{x0}}\sin w_x t)^2 - s_{y0}^{\ 2}(1+\frac{ds_y}{s_{y0}}\sin w_y t)^2) =$$

$$= \frac{1}{2}\frac{(a_{02}-a_{20})}{a_{00}}\{(d_x^{\ 2} - d_y^{\ 2} + s_{x0}^{\ 2} + s_{y0}^{\ 2}) + 2s_{x0}^{\ 2}\frac{ds_x}{s_{x0}}\sin w_x t - 2s_{y0}^{\ 2}\frac{ds_y}{s_{y0}}\sin w_y t +$$

$$+ s_{x0}^{\ 2}(\frac{ds_x}{s_{x0}}\sin w_x t)^2 + s_{y0}^{\ 2}(\frac{ds_y}{s_{y0}}\sin w_y t)^2\} = \frac{1}{2}\frac{(a_{02}-a_{20})}{a_{00}}\{(d_x^{\ 2} - d_y^{\ 2} + s_{x0}^{\ 2} + s_{y0}^{\ 2} + \frac{ds_x^{\ 2}}{2} + \frac{ds_y^{\ 2}}{2}) +$$

$$+ 2s_{x0}^{\ 2}\frac{ds_x}{s_{x0}}\sin w_x t - 2s_{y0}^{\ 2}\frac{ds_y}{s_{y0}}\sin w_y t - \frac{s_{y0}^{\ 2}}{2}(\frac{ds_y}{s_{y0}})^2\cos 2w_y t - s_{x0}^{\ 2}(\frac{ds_x}{s_{x0}})^2\cos 2w_x t\}$$

If horizontal and vertical quadrupole oscillations have different frequencies one can separate it using Fourier transformation of u_q. In horizontal plane amplitude of Fourier harmonic with ω_x is:

$$u_{qw_x} = \frac{(a_{02}-a_{20})}{a_{00}} s_{x0}^{\ 2}\frac{ds_x}{s_{x0}},$$

and amplitude of quadrupole oscillations is:

$$\frac{ds_{x}}{s_{x0}} = \frac{a_{00}}{(a_{02} - a_{20})} \frac{u_{qw_x}}{s_{x0}^{2}} = k \frac{u_{qw_x}}{s_{x0}^{2}}, \tag{3}$$

where $k = \dfrac{a_{00}}{a_{02} - a_{20}}$ characterize sensitivity of a particular BPM to quadrupole oscillations. It can be derived from measured map of BPM sensitivity or calculated from analytical model for BPM sensitivity. We used analytical model from [3]:

$$u_{t0}(x, y) = \frac{1}{pR}\left\{\frac{I_0(g\frac{\sqrt{x^2+y^2}}{R})}{f\, I_0(g)} + \sum_{m=1}^{\infty} \frac{2I_m(g\frac{\sqrt{x^2+y^2}}{R})}{m\, I_m(g)} \sin(mf)\cos(m(\frac{p}{2} - arctg(\frac{y}{x})))\right\}$$

where R is radius of BPM aperture, ϕ is half of subtended lobe angle, $g = \dfrac{2pR}{bgl}$ is relativistic correction for wavelength λ. Expansion to 2D series around point (x=0, y=0) up to 3-d order was calculated and coefficients for PSR BPM parameters (R=50.8mm, ϕ=17°) were found: a_{00}=.5919, a_{01}=.023, a_{02}=4.36e-4, a_{20}=-4.32e-4. It gives k=684mm^2. Note that equilibrium r.m.s. beam size at the BPM location should be known apriory. Results of computer simulations or measurements can be used. For further calculations we assume $\sigma_{x0} = \sigma_{y0}$ =10mm, then $\frac{ds_{x}}{s_{x0}} = 6.84 \cdot u_{qw_x}$. Similarly $\frac{ds_{y}}{s_{y0}} = 6.84 \cdot u_{qw_y}$.

III. Analysis of the signals from PSR BPM.

Signals from each of four lobes of PSR wide band strip line BPM wm41 were digitized by digital oscilloscope with 1ns interval and stored for off line analysis. Integration algorithm described in [4] was used to restore an original shape of the voltage pulse on the BPM electrodes. Sum of the signals from all four lobes is proportional to the beam current. Beam longitudinal profiles on several successive turns are shown in Fig.2(a,b).

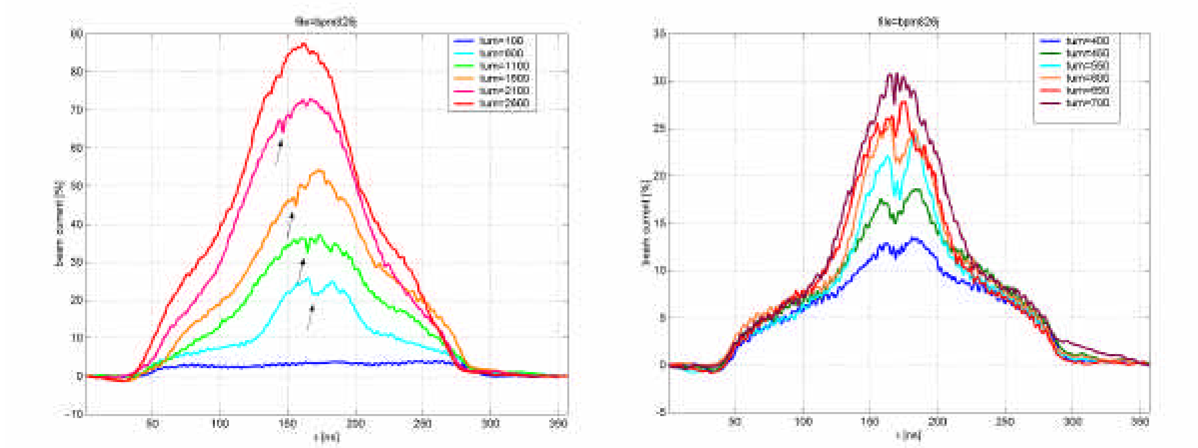

Figure 1. Longitudinal beam profiles at different turns. Amplitude is in % of maximum stored current.

Horizontal size of the plot of 357ns is equal to the period of one turn in the ring. There is a dip near beam profile maximum that appears just at the end of stacking and

y axis

is time inside each turn, x axis is turn number). The dip is well visible on the plot and

in very narrow zone near the maximum of the beam profile. This irregularity can indicate beam instability therefore the next step was to search for dipole and quadrupole

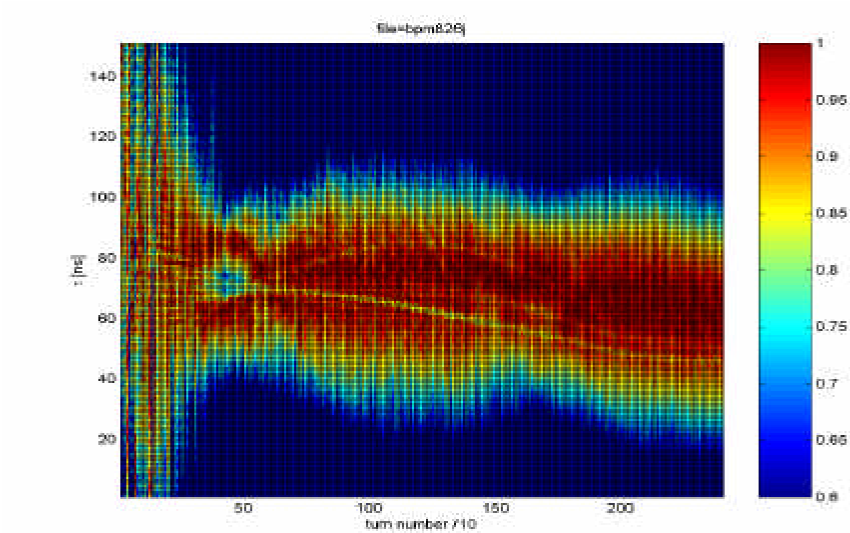

Figure 3. Color map of beam current from start of injection until extraction.

Transverse beam centroid position was calculated using difference over sum

oscillations of the several points with different longitudinal position τ are superimposed on the same graph). The spectrum of horizontal oscillations is similar (not

.25mm. In this case we can use (3) to find amplitude of coherent quadrupole transverse oscillations.

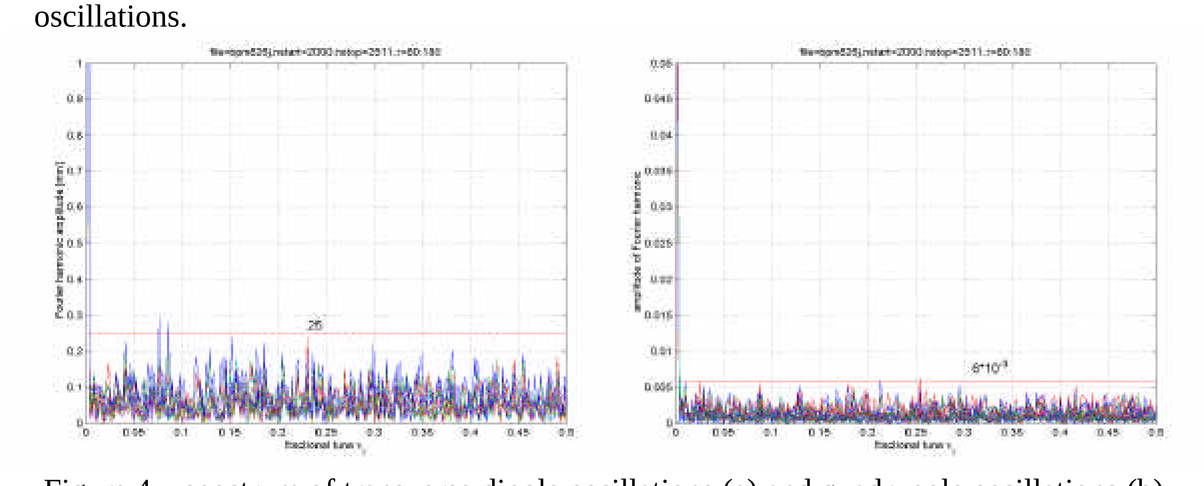

Figure 4.– spectrum of transverse dipole oscillations (a) and quadrupole oscillations (b).

The spectrum of u_{hv}/u_{Σ} is shown in Fig. 4b. Maximum of the amplitude of Fourier harmonics is less than .006 and using (3) we can set the limit for the amplitude of the transverse quadrupole oscillations: it is less than 4% of equilibrium beam size. We can conclude that there are no transverse dipole or quadrupole oscillations of significant amplitude.

Nevertheless on the Fourier spectrum of sum BPM signal shown in Fig.5 a well-defined peak near the double betatron frequency is observed. (FFT spectrum of the variation over many turns of the current at the point with longitudinal position τ is calculated and spectrums for several points with different τ are superimposed on one graph). Sum signal is not sensitive to dipole or quadrupole transverse oscillations so we can conclude that longitudinal quadrupole oscillations are observed in Fig.5.

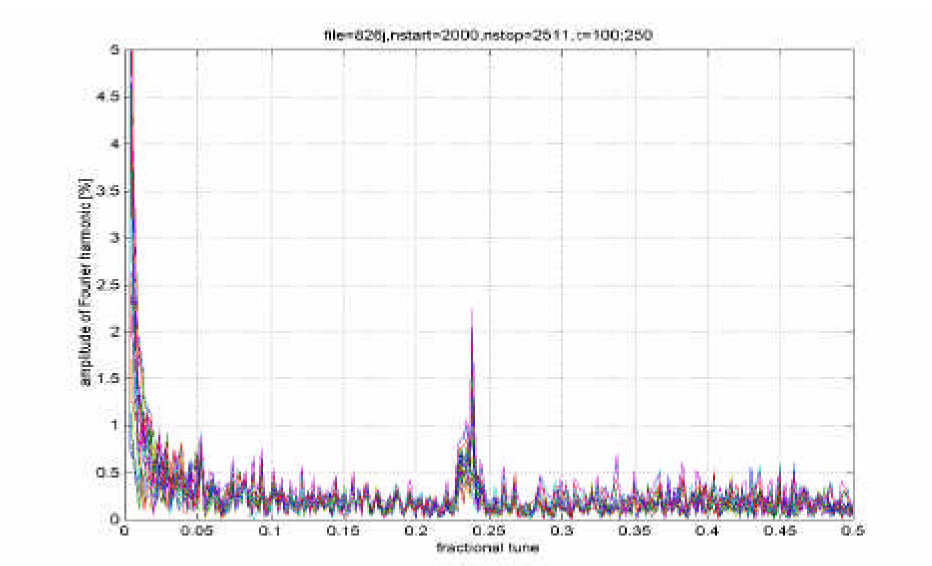

Figure 5. Spectrum of the sum BPM signal (in % of zero harmonic).

To verify this hypothesis signal from wall current monitor was analyzed because a wall current monitor is sensitive to longitudinal motion of the beam only. The color map of the beam current and Fourier spectrum are shown in Figs.6 a,b respectively. They are the same as data from BPM.

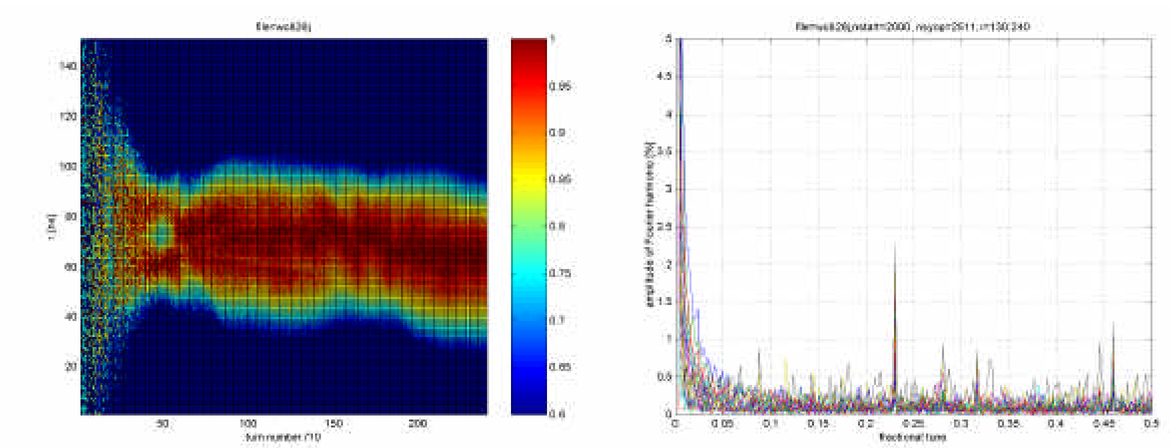

Figure 6. Color map of beam current measured by wall current monitor (a) and Fourier spectrum of wall current monitor signal (b) in % of zero harmonics.

The following plots illustrate some important features of observed longitudinal oscillations. In Fig.7 color map of absolute amplitude of the harmonic with $\nu \cong 2\nu_\beta$ is shown. We see that oscillations appear at the beginning of the injection and are localized in narrow zone near the maximum of the beam profile. Localization of the oscillations corresponds to density irregularity observed in Fig.3. This localization is strong argument against the possibility of interference from external source not related to the beam.

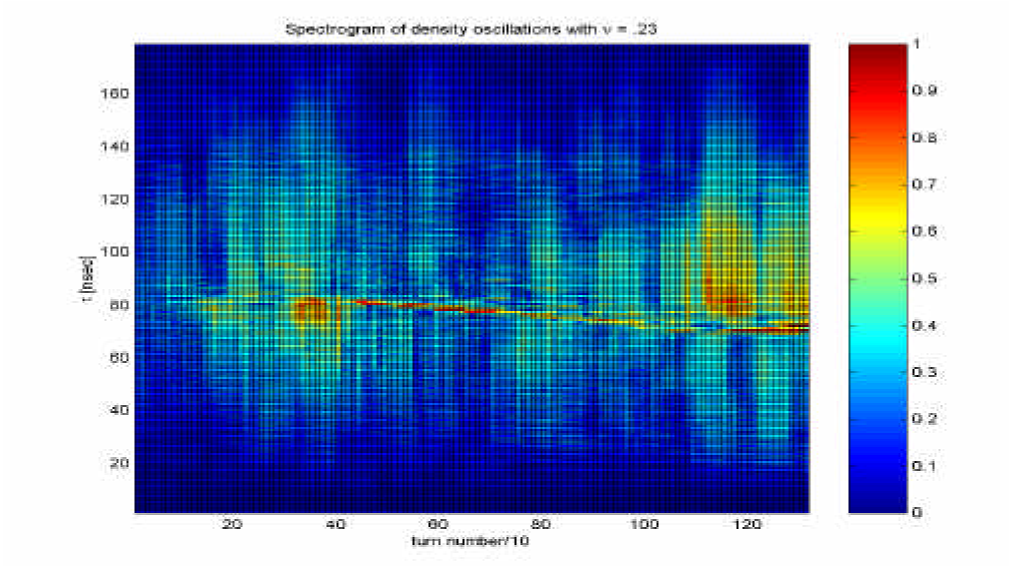

Figure 7. Color map of the amplitude of the harmonic near double betatron frequency.

The dependence of amplitude normalized on beam current upon turn number is shown in Fig.8a. The decreasing of normalized amplitude to the end of injection cycle can indicate that an observed phenomenon is a specific feature of the PSR injection scheme.

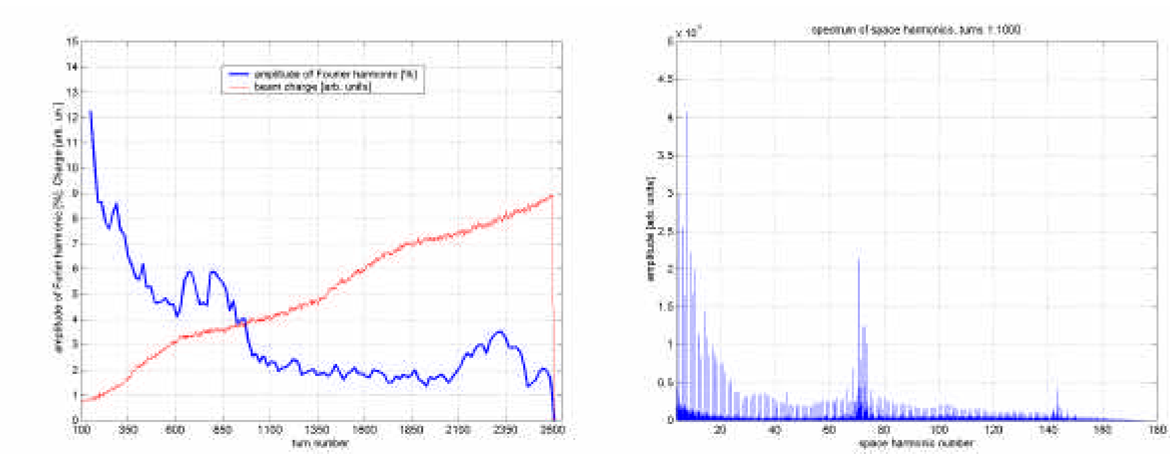

Figure 8. Dependence of the oscillation amplitude normalized on beam current (a) and spectrum of spatial harmonics (b)

The spectrum of spatial harmonics is shown in Fig.8b. The main spatial frequency is about 71 that together with revolution frequency of 2.8MHz gives absolute frequency of about 201MHz which is frequency of injected beam modulation. It is an additional indication that observed oscillations can be related to the injection pattern.

IV. Conclusion.

Analysis of BPM signals shows absence of dipole and quadrupole transverse oscillations of the beam. Longitudinal quadrupole oscillations are observed near the double betatron frequency most probably due to injection of high frequency modulated beam from the linac. Thought amplitude of the oscillations is small (near 1% of equilibrium value) oscillations are well measurable and can be used to check ability of a simulating code to resolve fine details of the storage ring performance.

V. Acknowledgements.

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VI. References.

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